

Hand out resources  
at the end of the PDF

# That was a Saturday!

John Brennan-Rhodes and Abi Bown

# Modular arithmetic

- This is essentially the remainder after division

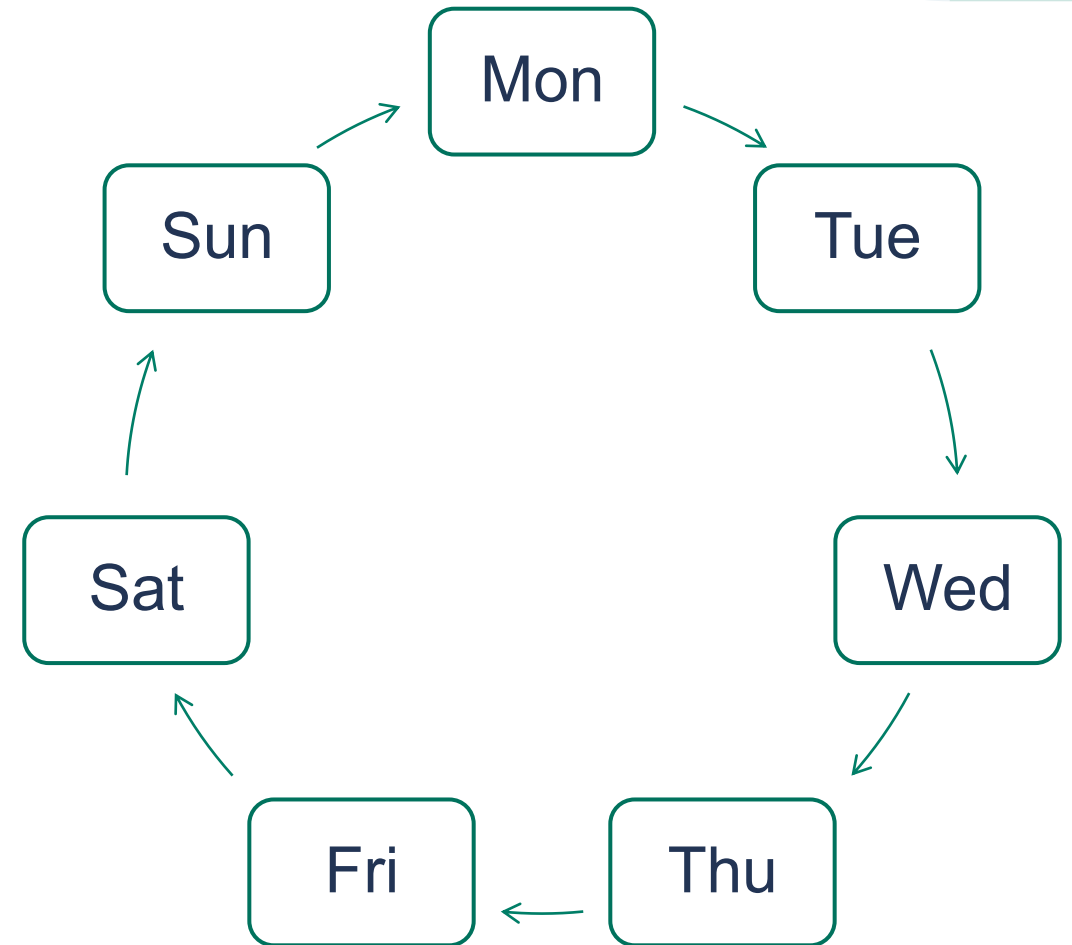
## Example

$$45 \div 7 = 6 \text{ remainder } 3$$

Can write

$$45 \equiv 3 \pmod{7}$$

- This is a very useful and has deeper topic in number theory





Because maths matters!



# Doomsdays!

# An observation about days

The days on the right, always fall on the same day of the week every year.

In 2026, this day is a Saturday.

2026's 'Doomsday' is a Saturday.

- Can you spot any patterns for some of the months?
- What about the other months?

**4/4, 6/6, 8/8, 10/10, 12/12**

January 3 (4 on leap)

February 28 (29 on leap)

March 14

April 4

May 9

June 6

July 11

August 8

September 5

October 10

November 7

December 12

# Suggested memory tools

| Month     | Doomsday        | Mnemonic         |
|-----------|-----------------|------------------|
| January   | 3 (4 on leap)   | 4 every four     |
| February  | 28 (29 on leap) | Last day         |
| March     | 14              | Pi day           |
| April     | 4               | "4/4"            |
| May       | 9               | "Working 9 to 5" |
| June      | 6               | "6/6"            |
| July      | 11              | "At the 7-11"    |
| August    | 8               | "8/8"            |
| September | 5               | "Working 9 to 5" |
| October   | 10              | "10/10"          |
| November  | 7               | "At the 7-11"    |
| December  | 12              | "12/12"          |

2026's Doomsday is **Saturday**.

Have at go at finding some days of the week in 2026 (not a leap year).

Maybe:

- Your birthday
- A family member's birthday
- October 14



# The history of the Doomsday rule

# Origin

## ‘Tomorrow is the day after Doomsday’

John Horton Conway - *Eureka*. October 1973. pp. 28–32.

- Can find the day of the week for any date in any year!
- Quite involved but Conway became very fast.



# Not the first rule!

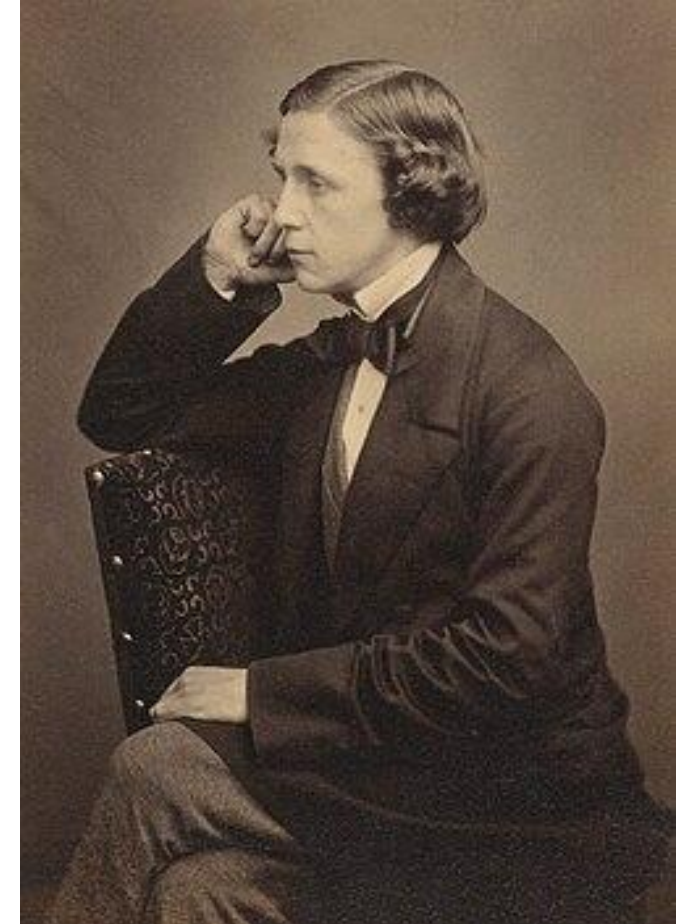
John Conway credits

“...I found the Doomsday rule by simplifying (almost beyond recognition!) a rule given by Lewis Carroll in *Nature*, 1872.”

## To Find the Day of the Week for any Given Date

Lewis Carroll - *Nature* (35,517) (1887)

“I am not a rapid computer myself, and as I find my average time for doing any such question is about 20 seconds, I have little doubt that a rapid computer would not need 15”



# An improvement/alternative

A simpler formula for the Doomsday of a given year!

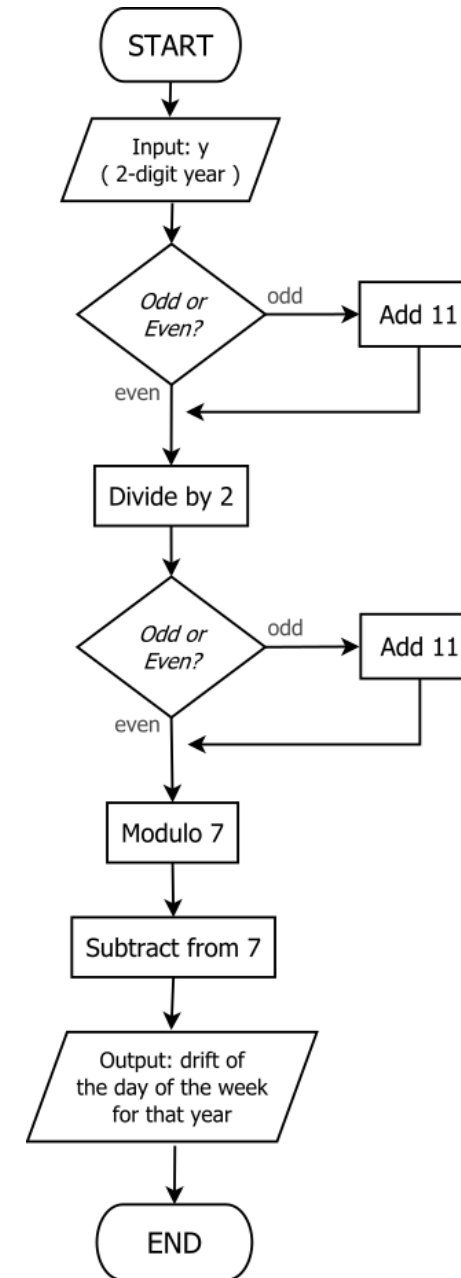
## Methods for Accelerating Conway's Doomsday Algorithm (part 2)

2010

Chamberlain Fong, Michael K. Walters

<https://arxiv.org/abs/1010.0765>

Other have also been found/suggested





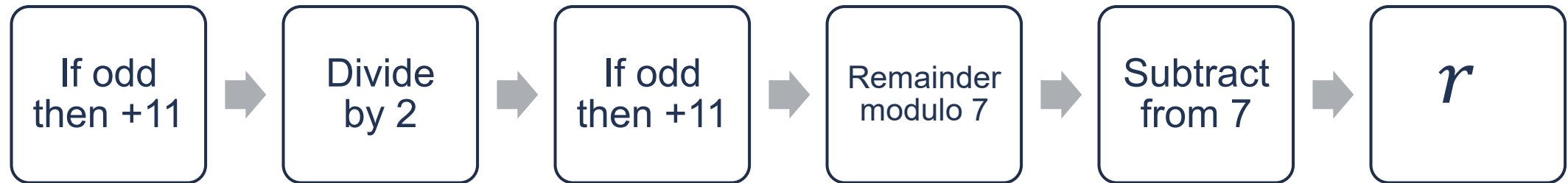
# Extending to any year

# Finding the Doomsday

**The century**  
(‘20’ for 2026)



**The year within the century**  
(‘26 for 2026)



**Anchor day +  $r$  = Doomsday for the year**

# Just remember the anchor day...

| Century       | Anchor day | Mnemonic                                                            |
|---------------|------------|---------------------------------------------------------------------|
| 1900–<br>1999 | Wednesday  | We-in-dis-day<br>(most living people were born in that century)     |
| 2000–<br>2099 | Tuesday    | Y-Tue-K or Twos-day<br>(Y2K was at the head of this century)        |
| 2100–<br>2199 | Sunday     | Twenty-one-day is Sunday<br>(2100 is the start of the next century) |

# Have a go - answers

| Event                                                                                               | Date                                     | Day of the week |
|-----------------------------------------------------------------------------------------------------|------------------------------------------|-----------------|
| Humans on the moon                                                                                  | 20 <sup>th</sup> July 1969               |                 |
| John Horton Conway's birthday                                                                       | 26 December 1937<br>(Died 11 April 2020) |                 |
| 1234567890 occurred on this day in Unix time                                                        | 13 February 2009                         |                 |
| The great fire of London starts                                                                     | 4 <sup>th</sup> September 1666           |                 |
| 13 <sup>th</sup> amendment abolishing slavery in the US                                             | 1 <sup>st</sup> February 1865            |                 |
| Florence Nightingale's birthday                                                                     | 12 <sup>th</sup> May 1820                |                 |
| The first time this religious event has happened on this date in the year in the Gregorian calendar | 29 <sup>th</sup> February 2096           |                 |

# Have a go - answers

| Event                                                                                               | Date                                     | Day of the week      |
|-----------------------------------------------------------------------------------------------------|------------------------------------------|----------------------|
| Humans on the moon                                                                                  | 20 <sup>th</sup> July 1969               | Sunday               |
| John Horton Conway's birthday                                                                       | 26 December 1937<br>(Died 11 April 2020) | Sunday<br>(Saturday) |
| 1234567890 occurred on this day in Unix time                                                        | 13 February 2009                         | Friday               |
| The great fire of London starts                                                                     | 4 <sup>th</sup> September 1666           | Saturday             |
| 13 <sup>th</sup> amendment abolishing slavery in the US                                             | 1 <sup>st</sup> February 1865            | Wednesday            |
| Florence Nightingale's birthday                                                                     | 12 <sup>th</sup> May 1820                | Friday               |
| The first time this religious event has happened on this date in the year in the Gregorian calendar | 29 <sup>th</sup> February 2096           | Wednesday            |



Because maths matters!

A large, solid green arrow pointing to the right, positioned to the left of the text.

# Why it works

# Dates within a year

Really the rule is split into two parts

4/4, 6/6, 8/8, 10/10, 12/12

Pairs of month April onwards have 61 days.

$$61 + 2 = 63 = 9 \times 7$$

Moving forward two months and two days is on the same day of the week (for some of the year – which?).

## The rest

Just nice ways to remember!

| Month     | Days | Pairs   |
|-----------|------|---------|
| January   | 31   |         |
| February  | 28   |         |
| March     | 31   |         |
| April     | 30   | 61 Days |
| May       | 31   |         |
| June      | 30   | 61 Days |
| July      | 31   |         |
| August    | 31   | 61 Days |
| September | 30   |         |
| October   | 31   | 61 Days |
| November  | 30   |         |
| December  | 31   |         |

# Doomsday of a year from the anchor

The formula from Conway.

If  $y$  is the last two digits of the year (i.e. '26' for '2026')

$$\left( \left\lfloor \frac{y}{12} \right\rfloor + y \bmod 12 + \left\lfloor \frac{y \bmod 12}{4} \right\rfloor \right) \bmod 7 + \text{anchor} = \text{Doomsday}$$

Where  $\lfloor x \rfloor$  is the 'floor of  $x$ ', the integer below.

Broadly this uses:

- That  $365 = 1 \bmod 7$  and  $366 = 2 \bmod 7$
- Difference between two days,  $y$  years apart is  $365y + \left\lfloor \frac{y}{4} \right\rfloor$

# Anchor year

... an exercise for the reader...





# The history of calendars

# Fun Fact 1

Which months of the year are named after Roman Gods?

# Fun Fact 1

January (Janus), March (Mars), May (Maia) and June (Juno) are named after Gods

## Fun Fact 2

Who created the original Roman calendar – how many months did it have?

## Fun Fact 2

The Roman calendar originated as a 10-month, 304-day lunar system created by Romulus

## Fun Fact 3

How many days were in a Roman Week?

## Fun Fact 3

Romans used an 8-day market week (A-H) rather than a 7-day week.

## Fun Fact 4

Which months were added to make a 12 month calendar, and when?

## Fun Fact 4

March was initially the first month, January and February were added around the 7<sup>th</sup> Century BC

## Fun Fact 5

What is the origin of the month name February

## Fun Fact 5

Even numbers were considered unlucky and so February (28 days) was devoted to the rites of purification (februa)

## Fun Fact 6

How did July and August get their current names?

## Fun Fact 6

July was originally named *Quintilis* (fifth) and August was *Sextilis* (sixth) before being renamed in honour of Julius Caesar and Augustus.

## Fun Fact 8

What is the rule for century leap years in our current calendar?

## Fun Fact 8

A leap day is added in years that ended in  
hundreds unless divisible by 400

## Fun Fact 7

Who introduced leap years into the Roman Calendar?

## Fun Fact 7

The leap years were added in 46 BC by Julius Ceasar

## Fun Fact 9

What change did Pope Gregory XIII make when introducing the Gregorian calendar?

## Fun Fact 9

To bring the calendar year back to the solar year, Pope Gregory XIII ordained that October 4 was to be followed by October 15 in 1582. Great Britain and the colonies adopted the Gregorian calendar in 1752.

## Fun Fact 10

What four countries don't use the Gregorian calendar?

## Fun Fact 9

The four countries that do not use the Gregorian calendar as their primary civil calendar are **Afghanistan, Iran, Ethiopia, and Nepal.**

# Calendar Investigations

- My mother is a hoarder and keeps old blank calendars and diaries – will she ever be able to use these again? When?
- 16/12/20 was a Pythagorean day (can you see why?) - what was the most recent and when is the next ... how many more will there be?
- After the French Revolution, France briefly used a new calendar based on a decimal system; 10 days a week, 10 hours a day, 100 minutes per hour and 100 seconds per minute. Investigate how this would work (how long is a second, how long is a year ...)

# Calendar Investigations

- A calendar year typically repeats with the exact same days and dates every **28 years** (assuming no break due to century years not div by 400). If a full 28-year cycle is not met, calendars can repeat in intervals of 6, 11, or 12 years depending on leap year positioning.
- The last one was 24/7/25, the next will be 24/10/26. I don't think there'll be any more!
- An hour was 144 conventional minutes (2.4 times as long as a conventional hour), a minute was 86.4 conventional seconds (44% longer than a conventional minute), and a second was 0.864 conventional seconds (13.6% shorter than a conventional second). The calendar consisted of twelve 30-day months, each divided into three 10-day 'weeks', plus five or six days at the end to fill out the balance of a year.



Because maths matters!



# Develop your own rule

# Do you like these?

| Month     | Doomsday        | Mnemonic         |
|-----------|-----------------|------------------|
| January   | 3 (4 on leap)   | 4 every four     |
| February  | 28 (29 on leap) | Last day         |
| March     | 14              | Pi day           |
| April     | 4               | "4/4"            |
| May       | 9               | "Working 9 to 5" |
| June      | 6               | "6/6"            |
| July      | 11              | "At the 7-11"    |
| August    | 8               | "8/8"            |
| September | 5               | "Working 9 to 5" |
| October   | 10              | "10/10"          |
| November  | 7               | "At the 7-11"    |
| December  | 12              | "12/12"          |

# Develop your own rule

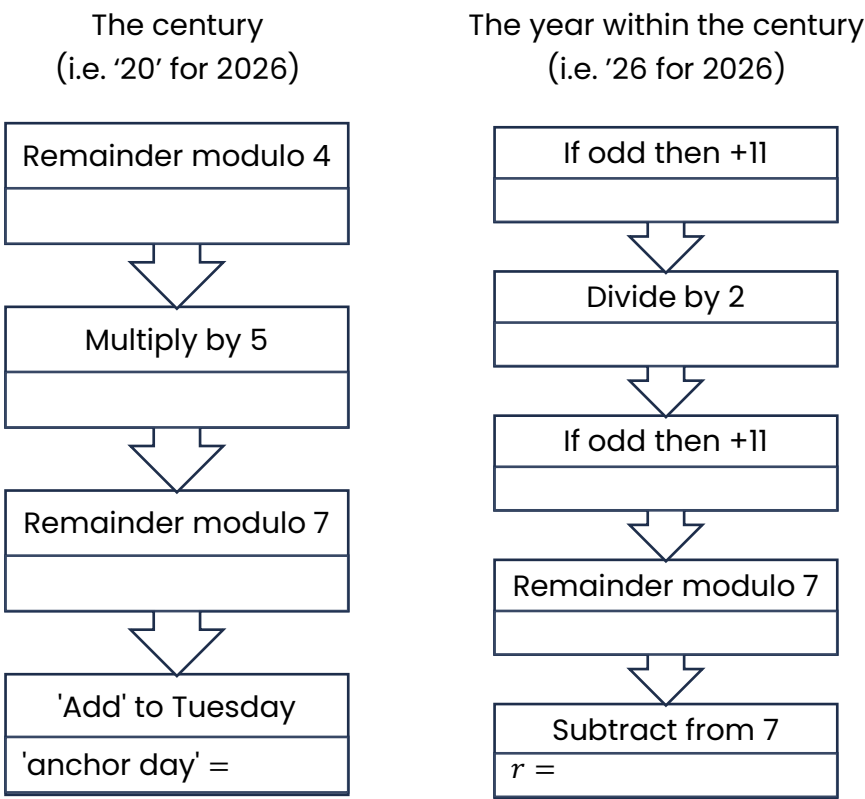
Look over the calendar and find the Doomsdays.

- Can you replace the rules for the months with your own version?
- Can you get quicker at the rule?



# The Doomsday Rule

|                  |  |
|------------------|--|
| Your chosen date |  |
|------------------|--|



|                                                    |  |
|----------------------------------------------------|--|
| The 'Doomsday' for the year = 'anchor day' + $r$ = |  |
|----------------------------------------------------|--|

Now find a day in that month that sits on the Doomsday.

| Month     | Doomsday        | Mnemonic         | Notes                            |
|-----------|-----------------|------------------|----------------------------------|
| January   | 3 (4 on leap)   | 4 every four     |                                  |
| February  | 28 (29 on leap) | Last day         | Multiples of 7 (+1 on leap year) |
| March     | 14              | Pi day           | Multiples of 7                   |
| April     | 4               | "4/4"            |                                  |
| May       | 9               | "Working 9 to 5" |                                  |
| June      | 6               | "6/6"            |                                  |
| July      | 11              | "At the 7-11"    |                                  |
| August    | 8               | "8/8"            |                                  |
| September | 5               | "Working 9 to 5" |                                  |
| October   | 10              | "10/10"          |                                  |
| November  | 7               | "At the 7-11"    |                                  |
| December  | 12              | "12/12"          |                                  |

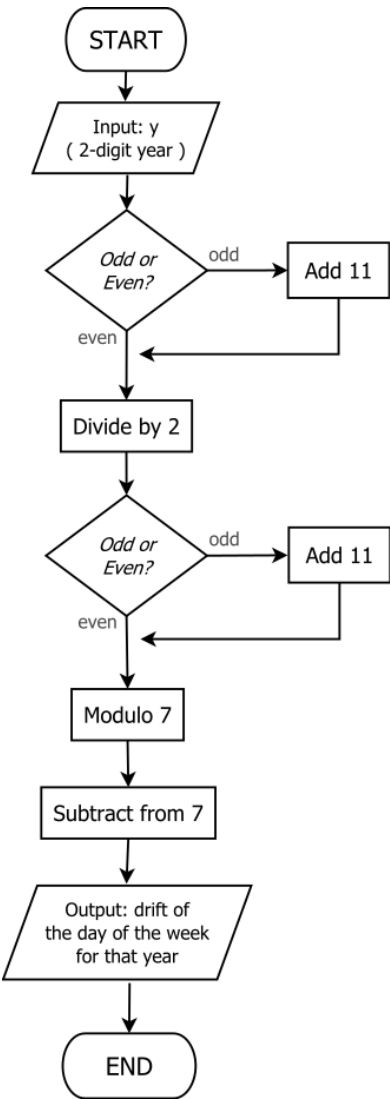
Work from that day to find the weekday for your chosen date  
E.g. if the Doomsday is Tuesday, and your date is 8<sup>th</sup> September  
5<sup>th</sup> September is a Tuesday  
So, 8<sup>th</sup> September is a "Tuesday +3" = Friday

|                           |  |
|---------------------------|--|
| Your chosen date was a... |  |
|---------------------------|--|

# Other presentations of the rule

## A flowchart for finding the Doomsday from the anchor day

(Wikipedia page – public domain)



## The rule in a single table

(Wikipedia page – public domain)

# The Doomsday Rule

| Weekday |        |         |           |          |        |          |
|---------|--------|---------|-----------|----------|--------|----------|
| Sunday  | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| 0       | 1      | 2       | 3         | 4        | 5      | 6        |

| Doomsday Month |          |           |         |          |          |  |
|----------------|----------|-----------|---------|----------|----------|--|
| January        | February | March     | April   | May      | June     |  |
| 31/32^         | 28/29^   | 7         | 4       | 9        | 6        |  |
| July           | August   | September | October | November | December |  |
| 11             | 8        | 5         | 10      | 7        | 12       |  |

^Leap Year

| Doomsday Century |         |         |         |
|------------------|---------|---------|---------|
| 1500             | 1600    | 1700    | 1800    |
| 1900             | 2000    | 2100    | 2200    |
| 2300             | 2400    | 2500    | 2600    |
| 3 (Wed)          | 2 (Tue) | 0 (Sun) | 5 (Fri) |

## Remembering the anchor day of years near now.

| Century   | Anchor day | Mnemonic                                                            |
|-----------|------------|---------------------------------------------------------------------|
| 1900–1999 | Wednesday  | We-in-dis-day<br>(most living people were born in that century)     |
| 2000–2099 | Tuesday    | Y-Tue-K or Twos-day<br>(Y2K was at the head of this century)        |
| 2100–2199 | Sunday     | Twenty-one-day is Sunday<br>(2100 is the start of the next century) |

# 2026

## January 01

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     | 1   | 2   | 3   | 4   |
| 5   | 6   | 7   | 8   | 9   | 10  | 11  |
| 12  | 13  | 14  | 15  | 16  | 17  | 18  |
| 19  | 20  | 21  | 22  | 23  | 24  | 25  |
| 26  | 27  | 28  | 29  | 30  | 31  |     |

## February 02

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     | 1   |
| 2   | 3   | 4   | 5   | 6   | 7   | 8   |
| 9   | 10  | 11  | 12  | 13  | 14  | 15  |
| 16  | 17  | 18  | 19  | 20  | 21  | 22  |
| 23  | 24  | 25  | 26  | 27  | 28  |     |

## March 03

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     | 1   |
| 2   | 3   | 4   | 5   | 6   | 7   | 8   |
| 9   | 10  | 11  | 12  | 13  | 14  | 15  |
| 16  | 17  | 18  | 19  | 20  | 21  | 22  |
| 23  | 24  | 25  | 26  | 27  | 28  | 29  |
| 30  | 31  |     |     |     |     |     |

## April 04

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     |     | 1   | 2   | 3   | 4   | 5   |
| 6   | 7   | 8   | 9   | 10  | 11  | 12  |
| 13  | 14  | 15  | 16  | 17  | 18  | 19  |
| 20  | 21  | 22  | 23  | 24  | 25  | 26  |
| 27  | 28  | 29  | 30  |     |     |     |

## May 05

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     | 1   | 2   | 3   |
| 4   | 5   | 6   | 7   | 8   | 9   | 10  |
| 11  | 12  | 13  | 14  | 15  | 16  | 17  |
| 18  | 19  | 20  | 21  | 22  | 23  | 24  |
| 25  | 26  | 27  | 28  | 29  | 30  | 31  |

## June 06

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 5   | 6   | 7   |
| 8   | 9   | 10  | 11  | 12  | 13  | 14  |
| 15  | 16  | 17  | 18  | 19  | 20  | 21  |
| 22  | 23  | 24  | 25  | 26  | 27  | 28  |
| 29  | 30  |     |     |     |     |     |

## July 07

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     |     | 1   | 2   | 3   | 4   | 5   |
| 6   | 7   | 8   | 9   | 10  | 11  | 12  |
| 13  | 14  | 15  | 16  | 17  | 18  | 19  |
| 20  | 21  | 22  | 23  | 24  | 25  | 26  |
| 27  | 28  | 29  | 30  | 31  |     |     |

## August 08

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     | 1   | 2   |
| 3   | 4   | 5   | 6   | 7   | 8   | 9   |
| 10  | 11  | 12  | 13  | 14  | 15  | 16  |
| 17  | 18  | 19  | 20  | 21  | 22  | 23  |
| 24  | 25  | 26  | 27  | 28  | 29  | 30  |
| 31  |     |     |     |     |     |     |

## September 09

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     | 1   | 2   | 3   | 4   | 5   | 6   |
| 7   | 8   | 9   | 10  | 11  | 12  | 13  |
| 14  | 15  | 16  | 17  | 18  | 19  | 20  |
| 21  | 22  | 23  | 24  | 25  | 26  | 27  |
| 28  | 29  | 30  |     |     |     |     |

## October 10

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     | 1   | 2   | 3   | 4   |
| 5   | 6   | 7   | 8   | 9   | 10  | 11  |
| 12  | 13  | 14  | 15  | 16  | 17  | 18  |
| 19  | 20  | 21  | 22  | 23  | 24  | 25  |
| 26  | 27  | 28  | 29  | 30  | 31  |     |

## November 11

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     |     |     |     |     |     | 1   |
| 2   | 3   | 4   | 5   | 6   | 7   | 8   |
| 9   | 10  | 11  | 12  | 13  | 14  | 15  |
| 16  | 17  | 18  | 19  | 20  | 21  | 22  |
| 23  | 24  | 25  | 26  | 27  | 28  | 29  |
| 30  |     |     |     |     |     |     |

## December 12

| Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|-----|-----|-----|-----|-----|-----|-----|
|     | 1   | 2   | 3   | 4   | 5   | 6   |
| 7   | 8   | 9   | 10  | 11  | 12  | 13  |
| 14  | 15  | 16  | 17  | 18  | 19  | 20  |
| 21  | 22  | 23  | 24  | 25  | 26  | 27  |
| 28  | 29  | 30  | 31  |     |     |     |

## Your Doomsday rule

| Month     | Doomsday | How to remember | Notes |
|-----------|----------|-----------------|-------|
| January   |          |                 |       |
| February  |          |                 |       |
| March     |          |                 |       |
| April     |          |                 |       |
| May       |          |                 |       |
| June      |          |                 |       |
| July      |          |                 |       |
| August    |          |                 |       |
| September |          |                 |       |
| October   |          |                 |       |
| November  |          |                 |       |
| December  |          |                 |       |