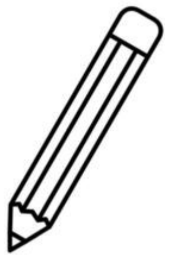
While you're waiting for the session to start, complete the questions below.

Compare your answers to others around you – did you use the same method?



1. Find the nth term of 5, 18, 35, 56, ...
2. Find the highest common factor of 180 and 225
3. Factorise $8x^2 - 77x - 30$
4. Divide $3x^3 + 8x^2 + 3x - 1$ by $(x - 2)$
5. Work out 84 - 27

Jo Morgan
@mathsjem
resourceaholic.com

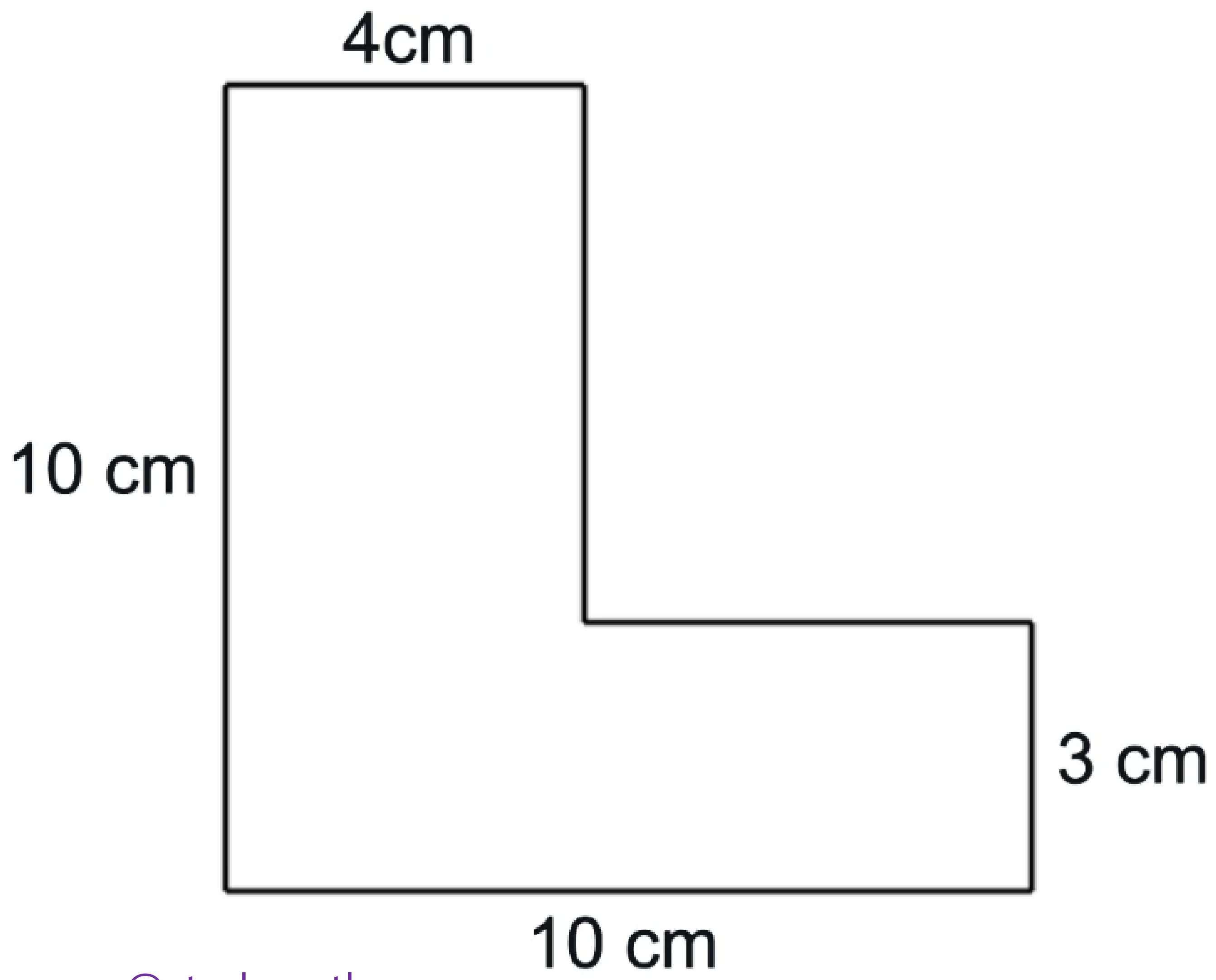
Let’s Talk About Methods

MEI Conference
July 2026

What do we mean by a method?

- An approach to solving a problem?
- A layout or structure?
- A mathematical procedure?

This compound shape is formed from two rectangles. Which calculation gives the area of the shape?



a) $4 \times 10 + 3 \times 6$

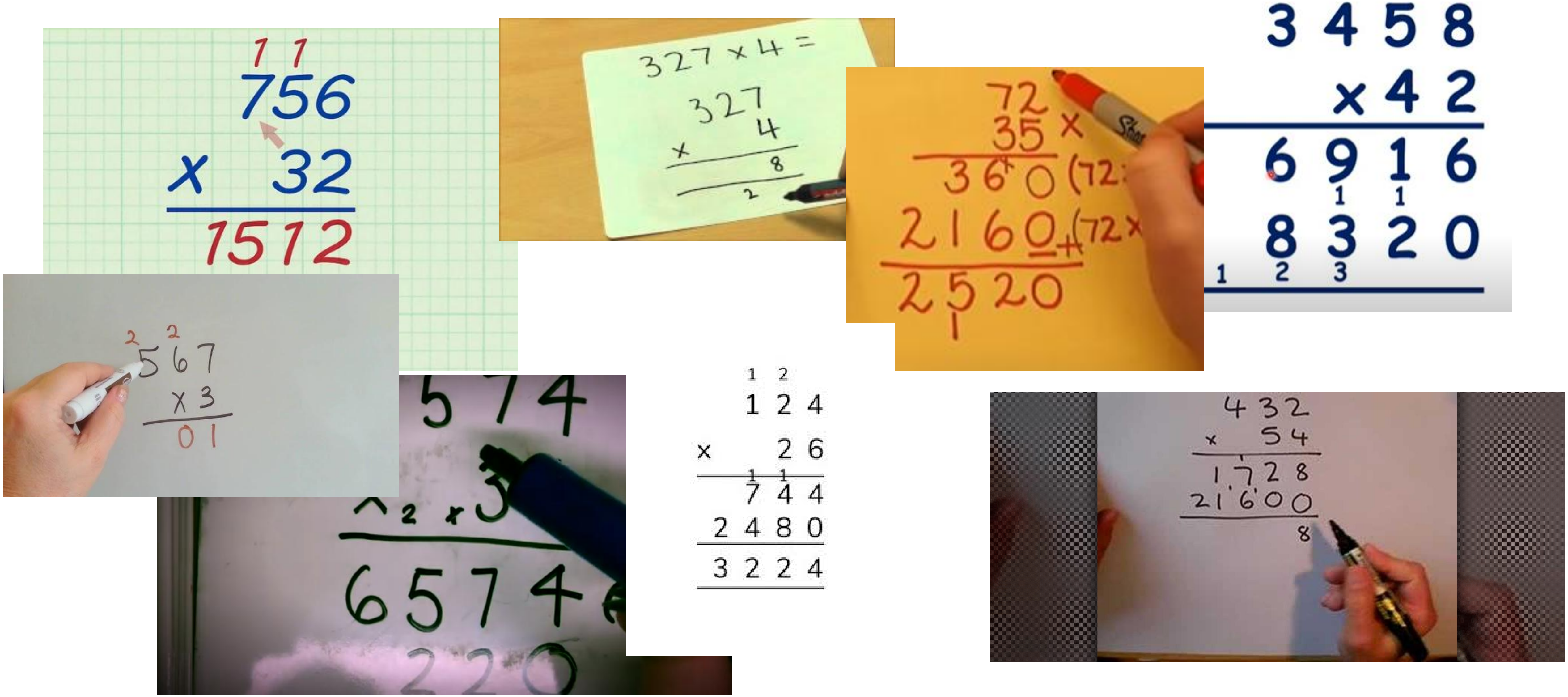
b) $3 \times 10 + 4 \times 7$

c) $10 \times 10 - 6 \times 7$

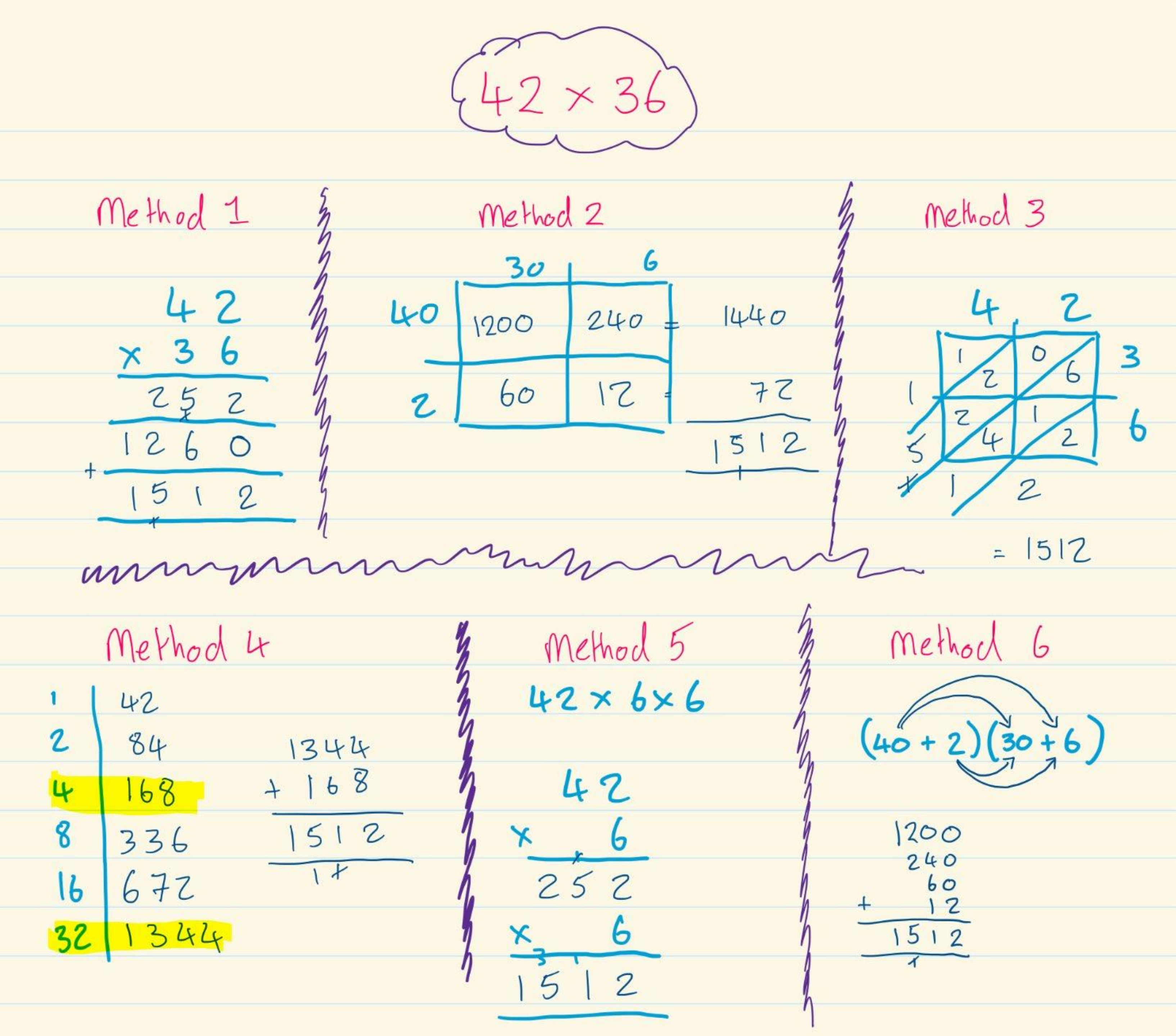
Source: @studymaths

What do we mean by a method?

- An approach to solving a problem?
- A layout or superficial structure?
- A mathematical procedure?



1 1
756
x 32
1512
327 x 4 =
327
x 4
8
2
72
35 x
360 (72
2160 (72 x
2520
3 4 5 8
x 4 2
6 9 1 6
8 3 2 0
2 2
567
x 3
01
6574
1 2
1 2 4
x 2 6
7 4 4
2 4 8 0
3 2 2 4
432
x 54
1728
21600
8



Source: @thecuriouscalc

a

Solve $\frac{10-x}{4} = 5x - 14$

$\times 4 \qquad \times 4$

$10 - x = 20x - 56$

$+x \qquad +x$

$10 = 21x - 56$

$+56 \qquad +56$

$66 = 21x$

$\div 21 \qquad \div 21$

$\frac{66}{21} = x$

b

Solve $\frac{10-x}{4} = 5x - 14$

$\times 4 \qquad \times 4$

$10 - x = 20x - 56$

$+x \qquad +x$

$10 = 21x - 56$

$+56 \qquad +56$

$66 = 21x$

$\div 21 \qquad \div 21$

$\frac{66}{21} = x$

c

Solve $\frac{10-x}{4} = 5x - 14$

$\times 4 \qquad \times 4$

$10 - x = 20x - 56$

$+x \qquad +x$

$10 = 21x - 56$

$+56 \qquad +56$

$66 = 21x$

$\div 21 \qquad \div 21$

$\frac{66}{21} = x$

D

Solve $\frac{10-x}{4} = 5x - 14$

$\times 4 \qquad \times 4$

$10 - x = 20x - 56$

$+x \qquad +x$

$10 = 21x - 56$

$+56 \qquad +56$

$66 = 21x$

$\div 21 \qquad \div 21$

$\frac{66}{21} = x$

@mathsplanner

Tweaks to minimise cognitive load

- Setting out...

vertical v. horizontal

eg $(3+2x)^4 = 1\,(3)^4(2x)^0$

$+\,4\,(3)^3(2x)^1$

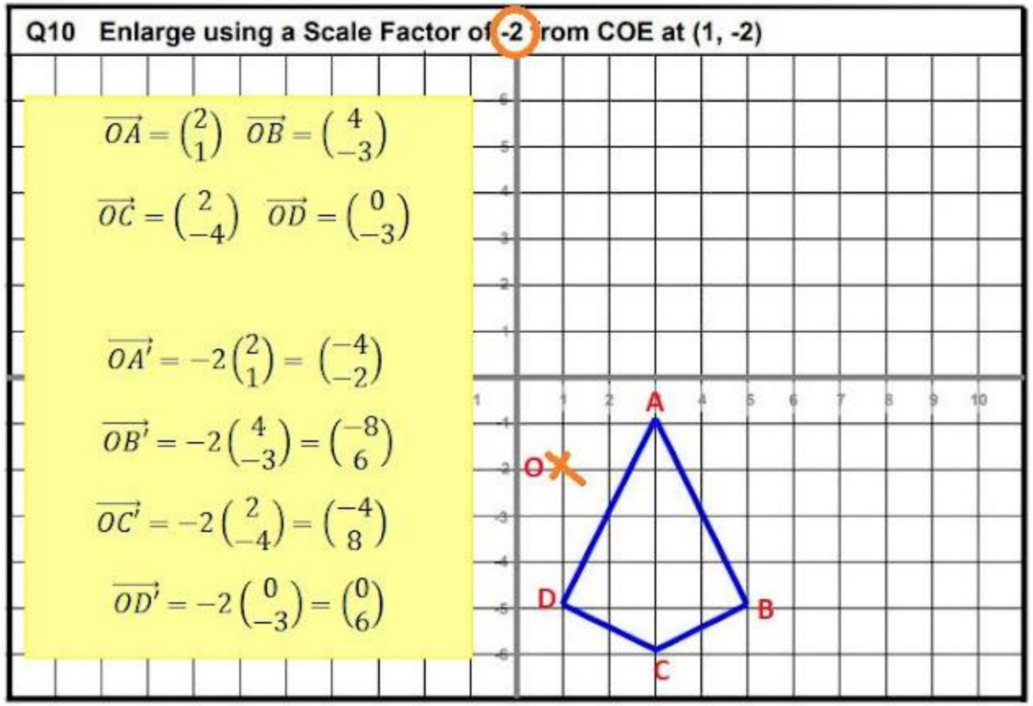
$+\,6\,(3)^2(2x)^2$

$+\,4\,(3)^1(2x)^3$

$+\,1\,(3)^0(2x)^4$

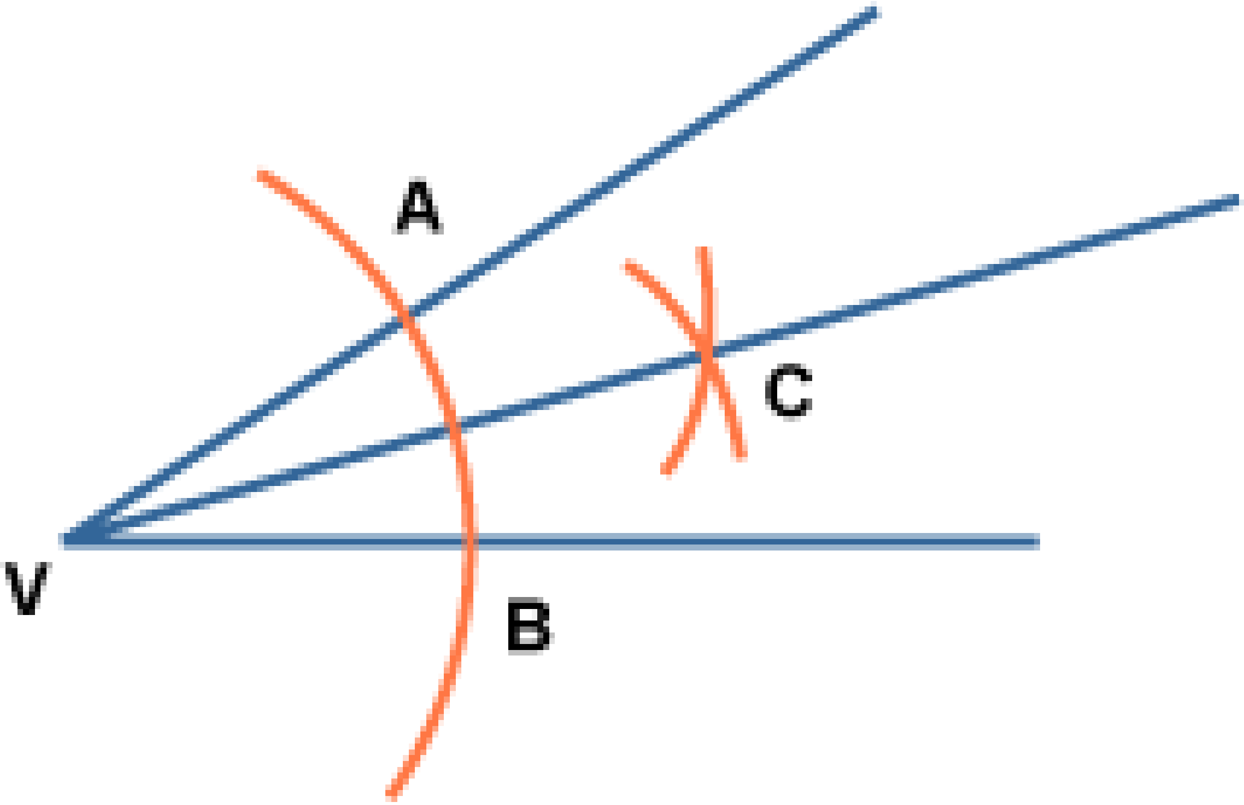
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & \\ & \end{bmatrix}$$

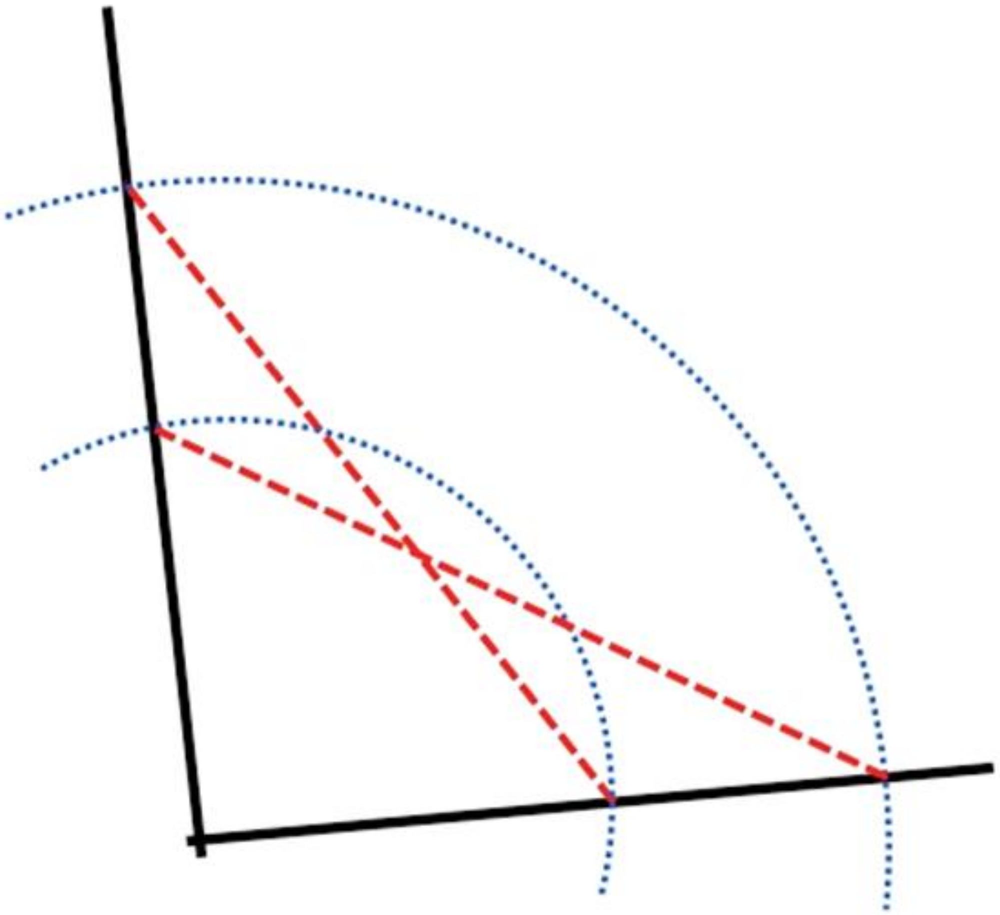
$$1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11 = 58$$



What do we mean by a method?

- An approach to solving a problem?
- A layout or superficial structure?
- A mathematical procedure?


A
C
V
B



A plumber charges £120 for a 4 hour job.

How much does the plumber charge for a 3 hour job?

Unitary Method

£120 for 4 hours

£30 for 1 hour

£90 for 3 hours

'Rule of Three'

$$\begin{array}{ccc} £120 & & 4 \\ & \times & \\ £x & & 3 \end{array}$$

£120 x 3 = 4x

There are many **international differences** in commonly used methods

$$(2x^4 + 3x^2 + 5x + 1) \div (x - 2)$$

Factorise $x^2 + 10x + 21$

$$x^2 + 10x + 21 = (x + 7)(x + 3)$$

$$x^2 + 10x + 21$$
$$= (x + 5)^2 - 25 + 21$$
$$= (x + 5)^2 - 4$$

$$= (x + 5 - 2)(x + 5 + 2)$$
$$= (x + 3)(x + 7)$$

$$x^2 + 4x - 32$$
$$= (x + 2)^2 - 4 - 32$$
$$= (x + 2)^2 - 36$$

$$= (x + 2 - 6)(x + 2 + 6)$$
$$= (x - 4)(x + 8)$$

Vincent Pantal🍩ni @panlepan · 18 Aug

Replying to @mathsjem

Funny that you say it's unusual, because that's how I taught it in Fr

Don't you complete the square that way ?

2 1 18

Arpitha Prasad @ArpithaPrasad6 · 18 Aug

Replying to @mathsjem

We did this very often actually. Basically every quadratic is of the form $x^2+2ax+b$, so you add and sub a^2 to get $(x+a)^2+b-a^2$. So we have (x+a+sqrt(b-a^2)) (x+a-sqrt(b-a^2)). I did not find this fiddly back when I was in school 🤔

1

Jo Morgan @mathsjem · 18 Aug

I just mean it's more fiddly than just writing down the factors (which normally takes seconds). What country did you learn it in?

1 1

Arpitha Prasad @ArpithaPrasad6 · 18 Aug

India. I preferred this and the quadratic formula to writing down the factors. I guess it makes a difference when the factors are unwieldy? To each their own I guess :)

1 1

Jo Morgan @mathsjem · 18 Aug

Absolutely. It's interesting that methods vary around the world. You're right, this has some advantages.

1

x^2+6x+8

$=(x+3)^2-9+8$

$=(x+3)^2-1^2$

$=(x+2)(x+4)$

2 6 19

Dr M Thornber

@DrMThornber

Replying to @mathsjem and @MrReddyMaths

I've had sixth formers who joined us from China, Japan and Singapore who all used this method as their first choice.

10:10 pm · 18 Aug 2019 · Tweetbot for iOS

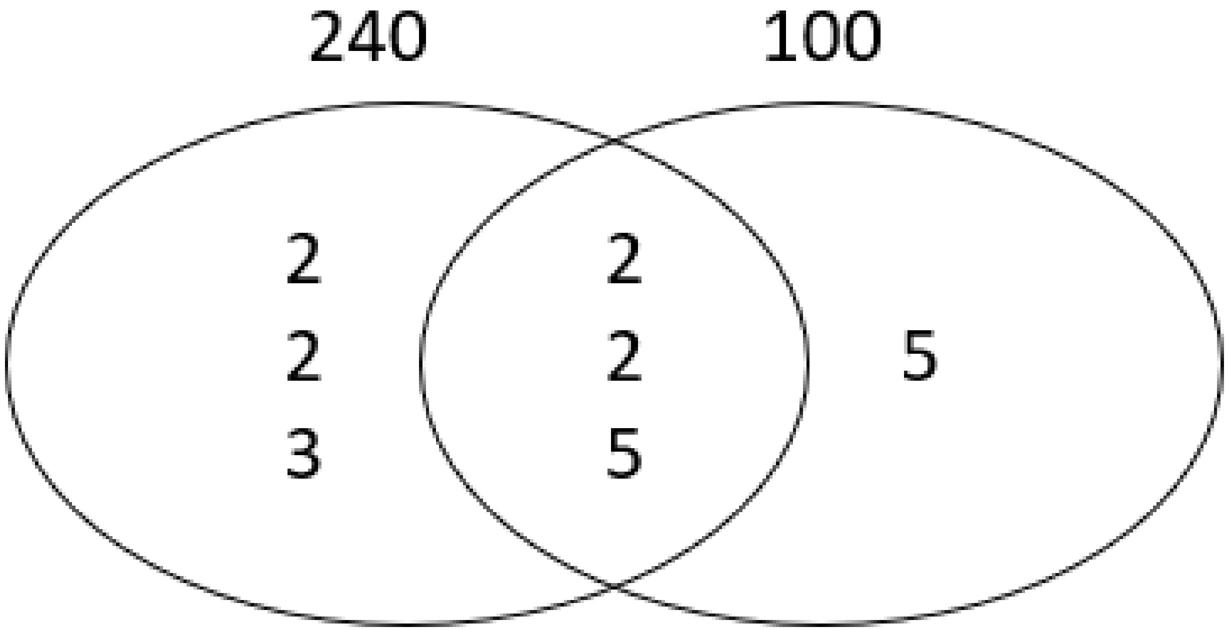
Simon Dube

@sim_dube

Replying to @mathsjem

We also teach this method in Quebec, Canada. A lot of students prefer this method to the others

2:30 am · 19 Aug 2019 · Twitter for iPhone



	240	100

This product is the required L.C.M.

We now apply this rule to find the L.C.M. of the numbers 64, 84, 96, and 216.

2)	64	84	96	216
2)	32	42	48	108
3)	16	21	24	54
2)	16	7	8	18
2)	8	7	4	9
2)	4	7	2	9
	2	7		9

The L.C.M. is $2\times2\times3\times2\times2\times2\times2\times7\times9=12{,}096$.

25. *Use of G.C.D. and L.C.M. in Solving Problems.*—Many

1945

702, 390 = 78
(Euclid's Algorithm)
HCF(390,702) = 2×3×13 = 78
13×3×2=78
HCF = 2×3×13 = 78
HCF = 78

Why care about methods?

- ✓**Controversial** ∴ interesting discussions.
- ✓We should explore methods with our pupils (they do in high performing jurisdictions) - it is **revealing and insightful** and leads to a **deeper understanding**.
- ✓Teachers should be able to offer a **toolkit** of approaches, not just their 'favourite' method.
- ✓This is what '**subject knowledge** development' looks like for experienced maths teachers.
- ✓We need to understand the methods our students are taught by other teachers (including online!) so we can provide the right **support** and **assess** work accurately.
- ✓It's **fun** to explore methods (teachers = curious mathematicians).

"Of course my method is the **best** method. My students always get their answers right"

What do we mean by ‘best’?

Easiest to teach

Most efficient

Most elegant

Most logical

Most applicable

Most widely used

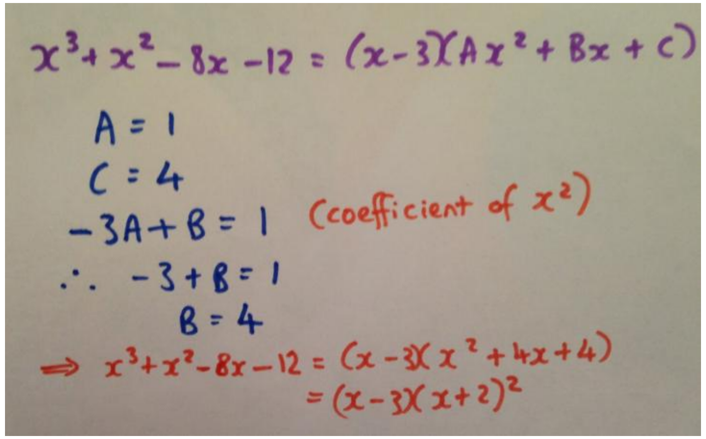
Easiest to remember

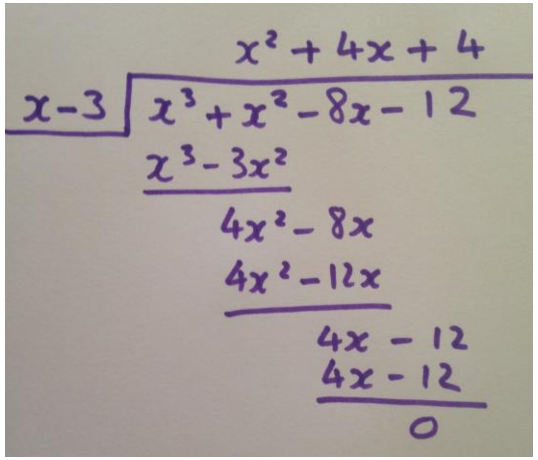
③

	$9x^2$	$9x$	5	
$3x$	$27x^3$	$27x^2$	$15x$	$+1$
-2	$-18x^2$	$-18x$	-10	R

④ $$\frac{27x^3+9x^2-3x-9}{3x-2}=9x^2+9x+5+\frac{1}{3x-2}$$

$(2x^4 + 3x^2 + 5x + 1) \div (x - 2)$

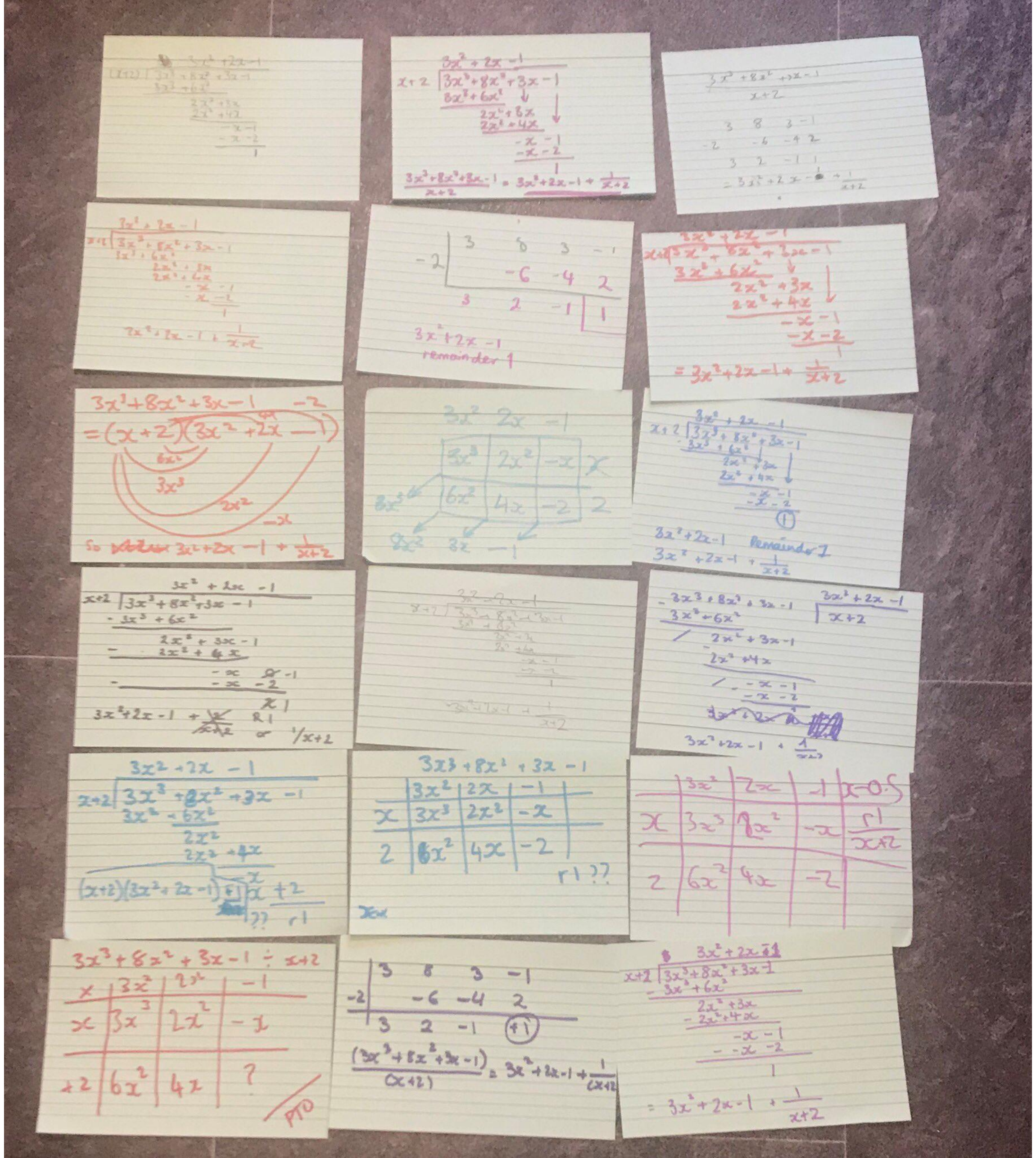




Divide $x^3 - 3x + 2$ by $x - 1$

$$x^2(x-1) \quad + x(x-1) \quad - 2(x-1)$$

$$= (x-1)(x^2 + x - 2)$$


3x²+2x-1
x+2 | 3x³+8x²+3x-1
3x³+6x²
2x²+3x
2x²+4x
-x-1
-x-2
1
(3x³+8x²+3x-1)/(x+2) = 3x²+2x-1 + 1/(x+2)
3x³+8x²+3x-1 / x+2
3 8 3 -1
-2 -6 -4 2
3 2 -1 1
3x²+2x-1
remainder 1
3x³+8x²+3x-1 ÷ -2
=(x+2)(3x²+2x-1)
6x²
3x³
2x²
-x
so 3x²+2x-1 + 1/(x+2)
3x² 2x -1
3x² 2x² -x x
6x² 4x -2 2
3x²+2x-1 Remainder 1
3x²+2x-1 + 1/(x+2)
3x²+2x-1 + 1/(x+2) R1 or 1/x+2
3x²+2x-1 + 1/(x+2)
3x² +2x -1
x+2 | 3x³ +8x² +3x -1
3x² +6x²
2x²
2x² +4x
-x
(x+2)(3x²+2x-1) +1 x +2
?? r1
3x³+8x²+3x-1
3x² 2x -1
x 3x³ 2x² -x
2 6x² 4x -2
r1 ??
3x² 2x -1 x-0.5
x 3x³ 2x² -x r1/(x+2)
2 6x² 4x -2
3x³+8x²+3x-1 ÷ x+2
x 3x² 2x -1
x 3x³ 2x² -1
+2 6x² 4x ?
PTO
3 8 3 -1
-2 -6 -4 2
3 2 -1 +1
(3x³+8x²+3x-1)/(x+2) = 3x²+2x-1 + 1/(x+2)
3x²+2x-1
x+2 | 3x³+8x²+3x-1
-3x³+6x²
2x²+3x
-2x²+4x
-x-1
-x-2
1
= 3x²+2x-1 + 1/(x+2)

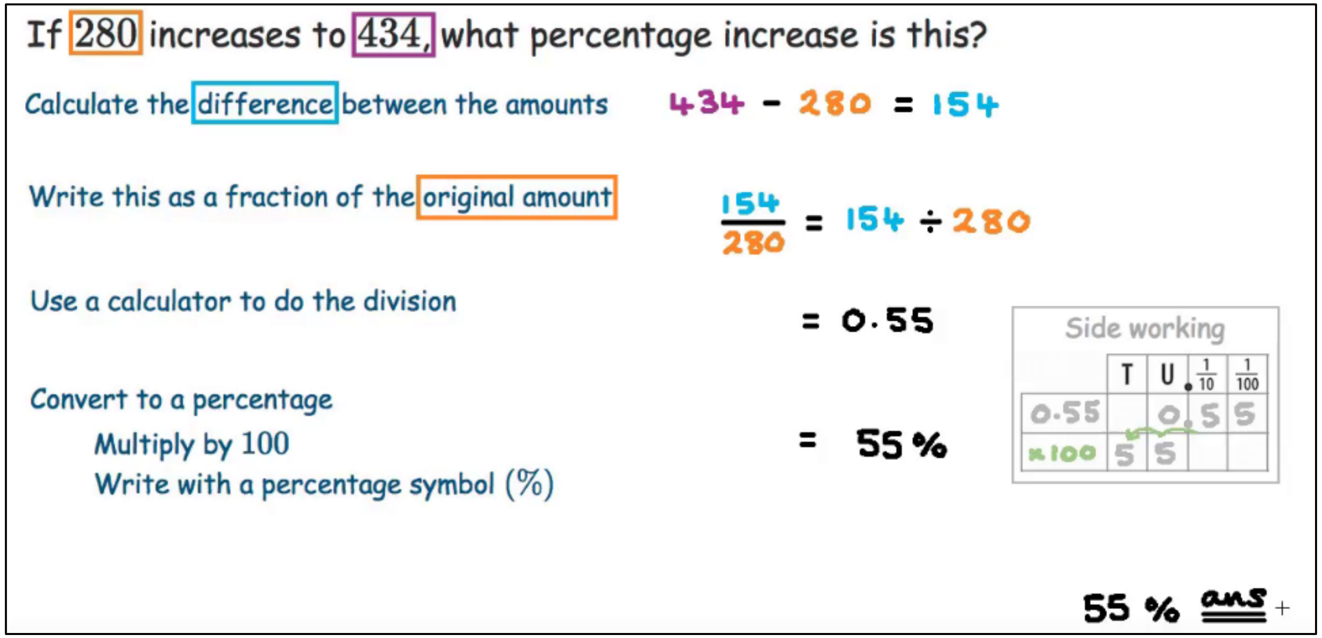
Some principles for method selection

1. Have an open mind – try new things.

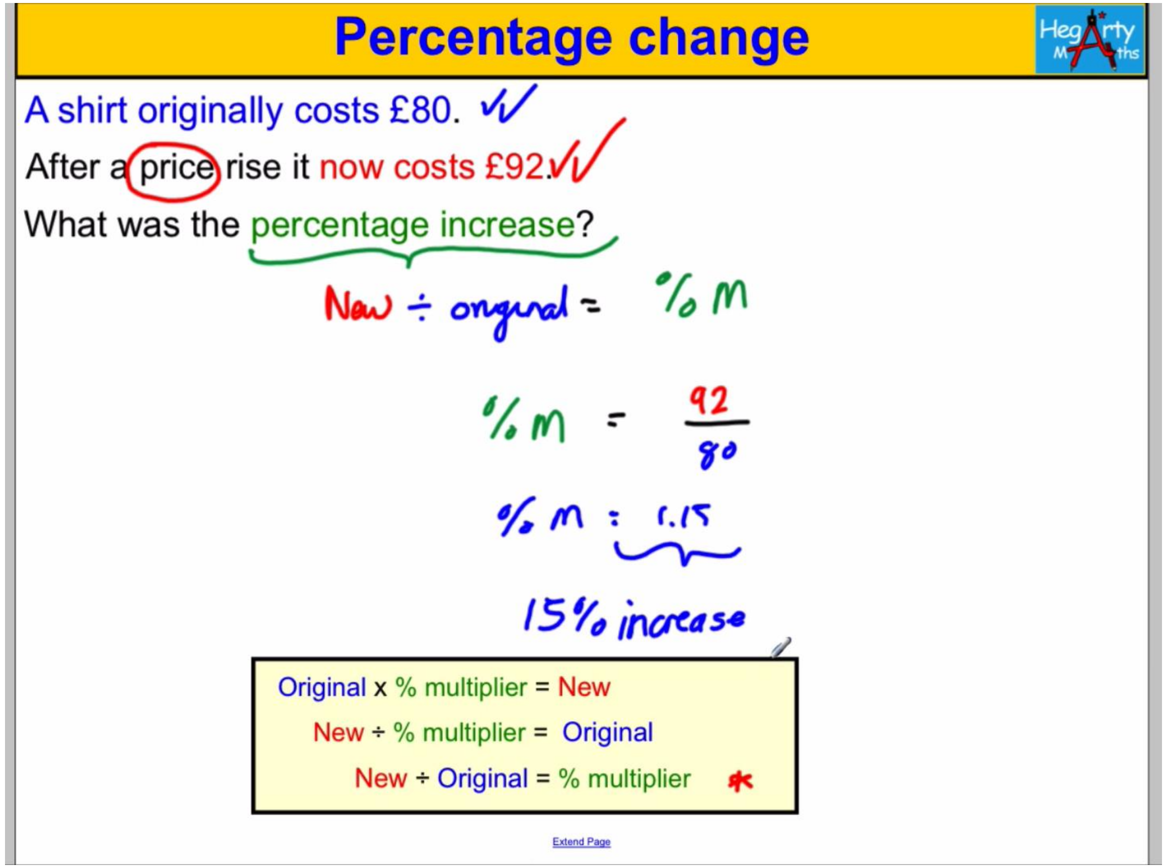
2. You can’t stop students from seeing other methods (from other teachers/tutors/parents/online) so we must always acknowledge that alternative methods exist.

3. Know your homework platforms!

4. Department CPD is very important.



Sparx



Hegarty

Expanding Double Brackets

1 Smiley Face/FOIL

$$(x+3)(x-5)$$

2 Two single Brackets

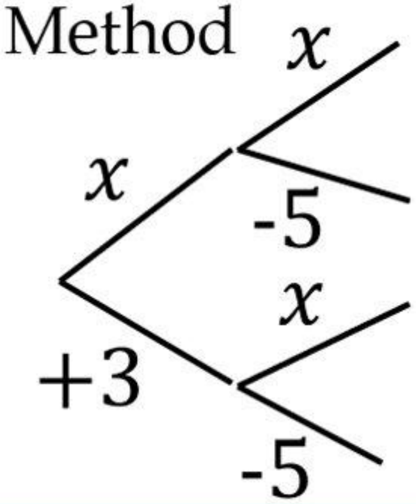
$$(x+3)(x-5)$$

$$= x(x-5) + 3(x-5)$$

3 Grid Method

	x	$+3$
x		
-5		

4 Tree Method



nd and simplify $(x + 8)(x - 10)$

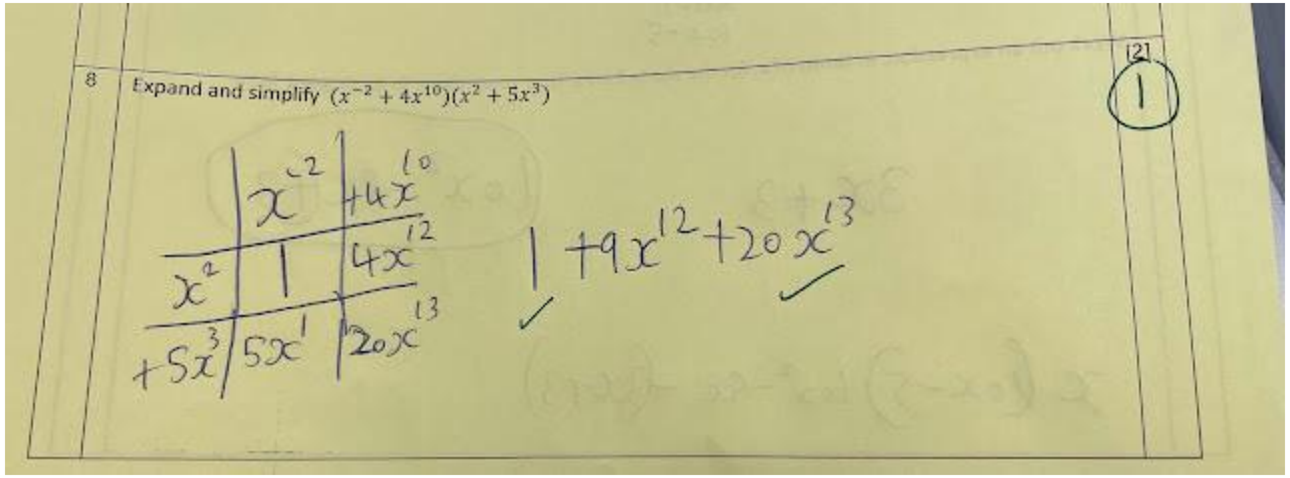
$x^2 - 10x + 8x - 2$ ✗

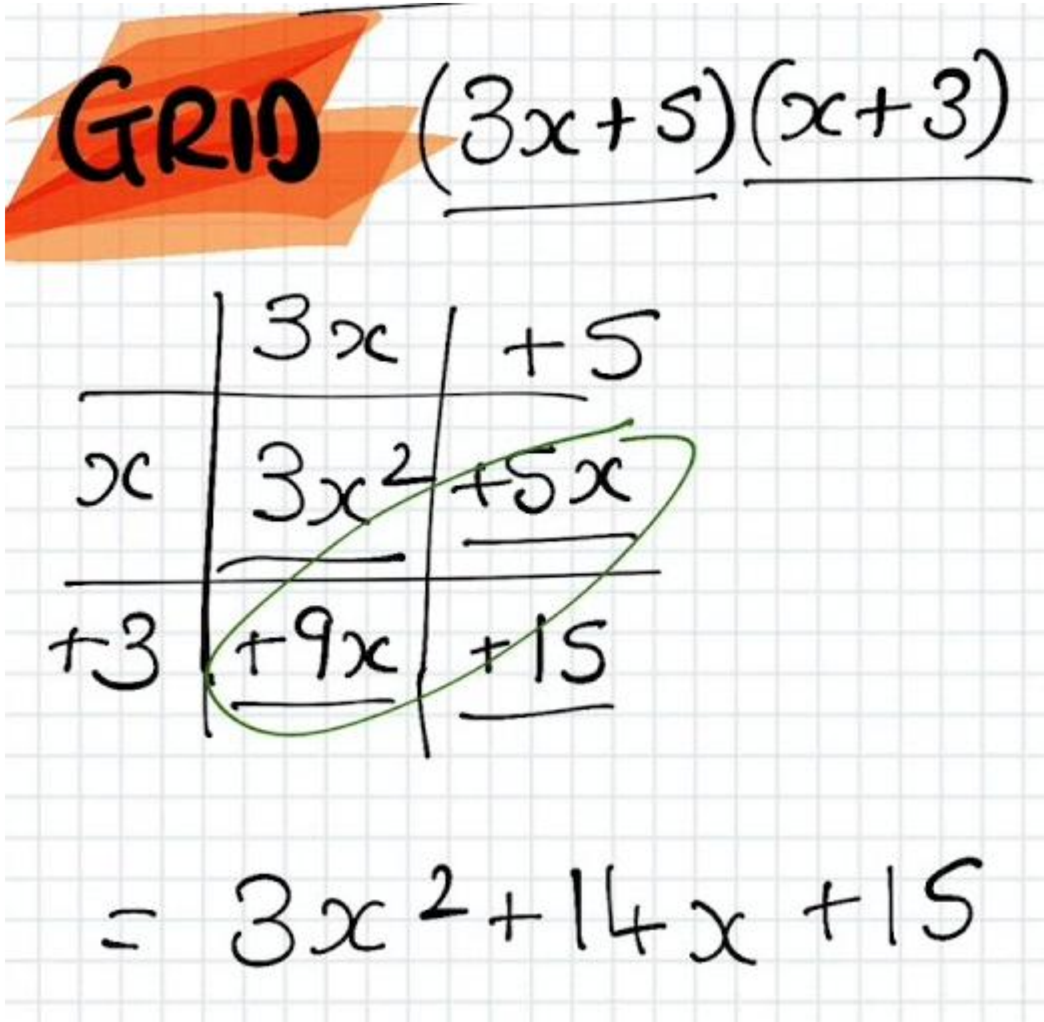
$x^2 - 2x - 2$

	x	$+8$
x	x^2	$+8x$
-10	-10	-2

(2 marks

1





'sausage/balloon/peanut'

Distributive Method

$$
\begin{aligned}
&(4x+3)(2x+10)\\
&= 4x(2x+10)+3(2x+10)\\
&= 8x^2+40x+6x+30\\
&= 8x^2+46x+30
\end{aligned}
$$

Factorising non-monics

Factorise $15x^2 + 26x + 8$

"The beginner will frequently find that is it not easy to select the proper factors at the first trial. Practice alone will enable him to detect at a glance whether any pair he has chosen will combine so as to give the correct coefficients of the expression to be resolved". (1885)

Suppose that we wish to factorise $15x^2 + 26x + 8$.

The first terms in the brackets may be

$$(15x \quad)(x \quad) \quad \text{or} \quad (5x \quad)(3x \quad)$$

All the signs must be $+$.

The brackets may end with 8 and 1, 4 and 2, 2 and 4, or 1 and 8.

Try each in turn, until the correct middle term is found.

$(15x + 8)(x + 1)$	middle term $23x$, incorrect
$(15x + 4)(x + 2)$	middle term $34x$, incorrect
$(15x + 2)(x + 4)$	middle term $62x$, incorrect
$(15x + 1)(x + 8)$	middle term $121x$, incorrect
$(5x + 8)(3x + 1)$	middle term $29x$, incorrect
$(5x + 4)(3x + 2)$	middle term $22x$, incorrect
$(5x + 2)(3x + 4)$	middle term $26x$, *correct*

$$\therefore \quad 15x^2 + 26x + 8 = (5x + 2)(3x + 4)$$

After a little practice, you should be able to find the factors without going into so much detail.

Factorise $6x^2 + 17x + 12$ by inspection ('guess and check').

$$(3x + \quad)(2x + \quad)$$

$(6x + 1)(x + 12)$	$(6x + 1)(x + 12)$
$(6x + 12)(x + 1)$	$\cancel{(6x + 12)(x + 1)}$
$(6x + 2)(x + 6)$	$\cancel{(6x + 2)(x + 6)}$
$(6x + 6)(x + 2)$	$\cancel{(6x + 6)(x + 2)}$
$(6x + 3)(x + 4)$	$\cancel{(6x + 3)(x + 4)}$
$(6x + 4)(x + 3)$	$\cancel{(6x + 4)(x + 3)}$
$(2x + 1)(3x + 12)$	$\cancel{(2x + 1)(3x + 12)}$
$(2x + 12)(3x + 1)$	$\cancel{(2x + 12)(3x + 1)}$
$(2x + 2)(3x + 6)$	$\cancel{(2x + 2)(3x + 6)}$
$(2x + 6)(3x + 2)$	$\cancel{(2x + 6)(3x + 2)}$
$(2x + 3)(3x + 4)$	$(2x + 3)(3x + 4)$
$(2x + 4)(3x + 3)$	$\cancel{(2x + 4)(3x + 3)}$

Factorise $6x^2 + 17x + 12$ by grouping.

$ac = 72$

Factors of 72

1, 72
2, 36
3, 24
4, 18
6, 12
8, 9

Split:

$$6x^2 + 17x + 12 = 6x^2 + 8x + 9x + 12$$

Factorise each half:

$$6x^2 + 8x + 9x + 12 = 2x(3x + 4) + 3(3x + 4)$$

Collect (factorise):

$$(3x + 4)(2x + 3)$$

Issues

✓ Retention (highly procedural)
✓ Detachment from prerequisite skills

MAFF = Multiply, add, factorise, factorise

19b : 0

19 (b) Factorise $8x^2 - 18x - 35$ **[2 marks]**

280 — 14 20

1 280
2 140
4 70
5 56
7 40
8 35
10 28

$8 \times 35 = 280$ ~~14~~ 28, ~~-5~~ 10

$\cancel{8x^2 - 10x - 28x - 5}$ $8x^2 - 28x + 10x - 35$

~~$x(8x - 28)$ | $(\ \ )$~~

$8x(x - 3.5)$ | $10(x - 3.5)$

Answer $8x(x-3.5)\ 10(x-35)$

19 (b) Factorise $8x^2 - 18x - 35$ **[2 marks]**

$8 \times -35 = -280$ $-280 = -28 \times 10$

$\cancel{8x^2}$ $8x^2 + 10x$ | $-28x - 35$

$2x(4x + 5)$ | $-7(4x + 5)$

$(4x + 5)(2x - 7)$

Answer $(4x + 5)(2x - 7)$

Factorising non-monics

Factorise $15x^2 + 26x + 8$

Grid Method

Factorise $8x^2 - 18x - 35$

[2 marks]

$8x^2 - 18x - 35$

$8 \times 35 = 280$

2 140
4 70
5 56
7 40
8 35
20 14
-10 -28

	x	18
$8x$	$8x^2$	-10 -28x
-28x	28x -10x	-280

Answer $(8x - 28x)(\cancel{-10x}\ \cancel{+280})$

19 (b) Factorise $8x^2 - 18x - 35$

[2 marks]

-7 5
7 -5
35 -1
-35 1

$(4x+5)(2x-7)$

	$4x$	5
$2x$	$8x^2$	$10x$
-7	-28x	-35

$8x^2 - 18x - 35$

Answer $(4x+5)(2x-7)$

Factorise $6x^2 + 17x + 12$ by Lyszkowski's method.

$ac = 72$

<u>Factors of 72</u>

1, 72

2, 36

3, 24

4, 18

6, 12

8, 9

$$\frac{(6x+8)(6x+9)}{6}$$

$$= \frac{2(3x+4)3(2x+3)}{6}$$

$$= (3x+4)(2x+3)$$

7.7 Nix: Slide and Divide, aka Throw the Football

Because:

$$2x^2 - 5x - 12$$

slide

$x^2 - 5x - 24$

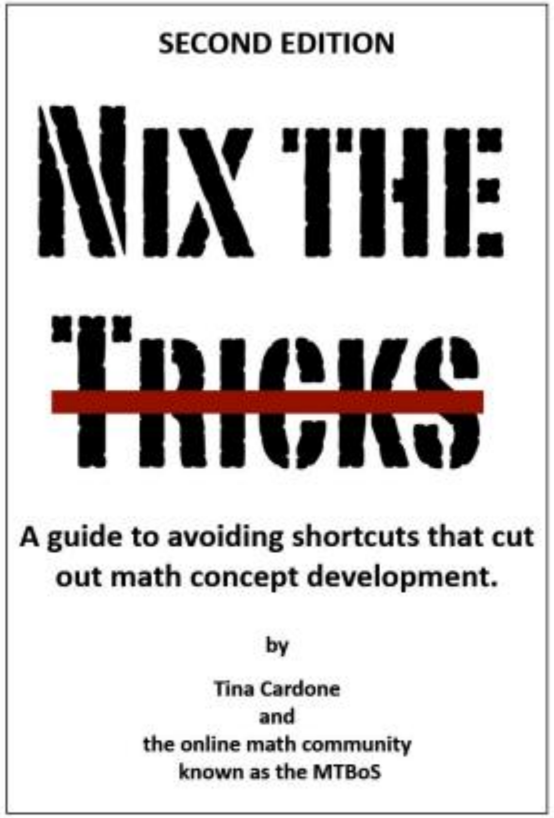
$(x - 8)(x + 3)$

now divide:

$(x - 8/2)(x + 3/2)$

$(x - 4)(2x + 3)$

The middle steps are not equivalent to the original expression so students cannot switch to an alternate method partway through. Either students are led to believe that $2x^2 - 5x - 12 = x^2 - 5x - 24$ (which is false) or they know it is not equal and they are forced to blindly trust that the false equivalence will somehow result in a correct answer. Do not ask for students' faith, show them that math makes sense.



AQA June 2024

19b : 0

19 (b) Factorise $8x^2 - 18x - 35$

[2 marks]

-280

$\frac{-28}{8}$ $\frac{10}{8}$

-18

$(8x - 28)(8x + 10)$

Answer $(8x-28)(8x+10)$

Factorise $6x^2 + 17x + 12$ by the monic method.

$$y = 6x^2 + 17x + 12$$

Where's the research...?

$$\begin{array}{r} 8\quad 4 \\ -\quad 2\quad 7 \\ \hline \end{array}$$

$$
\begin{array}{rrr}
 & \overset{7}{\cancel{8}} & {}^{1}4 \\
- & 2 & 7 \\
\hline
\end{array}
$$

$$\begin{array}{rcc} & 8 & {}^{1}4 \\ - & {}^{3}\cancel{2} & 7 \\ \hline \end{array}$$

$$\begin{array}{rrr} & {}^{7}\cancel{8} & {}^{1}4 \\ - & 2 & 7 \\ \hline \end{array}$$

Decomposition

Re-write 84 as 70 + 14

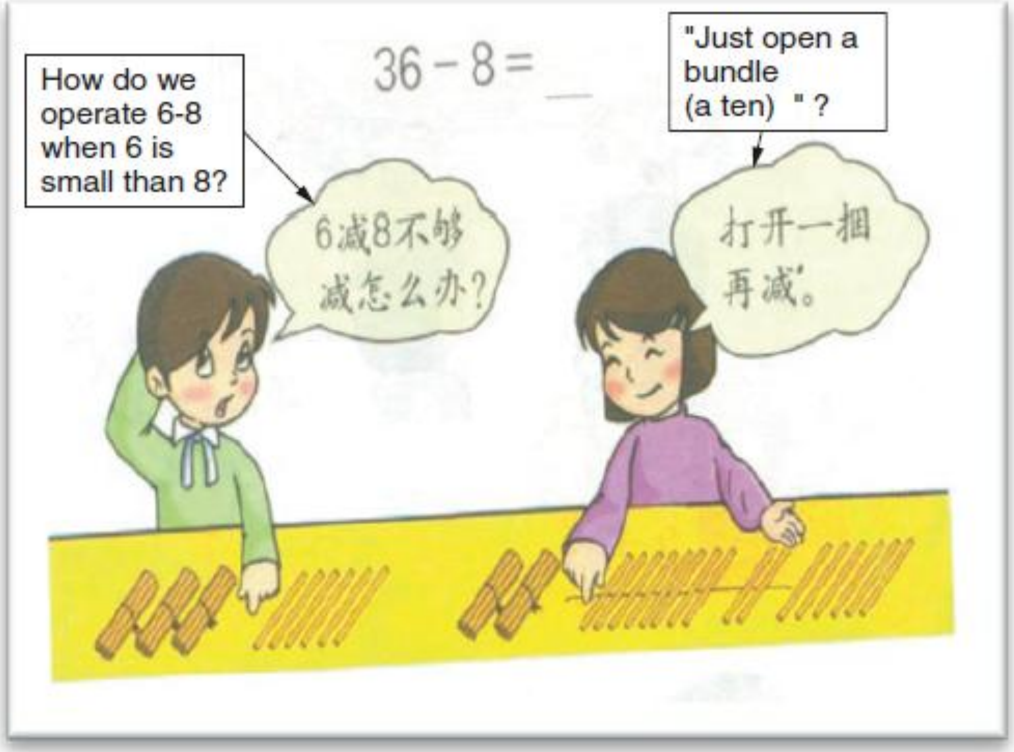


Fig. 3.8 Subtraction in the Chinese textbook (Mathematics Textbook Developer Group for Elementary School, 2005, vol. 2, p. 68.)

$$\begin{array}{rrr} & 8 & {}^{1}4 \\ - & {}^{3}\cancel{2} & 7 \\ \hline \end{array}$$

Equal Addition

84 – 27 becomes 94 - 37

Mrs Noble
@gemteaches

Replying to @mathsjem

I finally understand the way I was taught subtraction in the 80s. I was taught the equal addition method as a trick with no explanation - we were taught it as a "jimmy to the top and a jimmy to the bottom" - I had no idea who or what jimmy was 😂

7:38 pm · 20 Dec 2019 · Twitter for iPhone

$$\begin{array}{rcc} & {}^{7}\cancel{8} & {}^{1}4 \\ - & 2 & 7 \\ \hline \end{array}$$

Decomposition
(my method)
Re-write 84 as 70 + 14

$$\begin{array}{rcc} & 8 & {}^{1}4 \\ - & {}^{3}\cancel{2} & 7 \\ \hline \end{array}$$

Equal Addition
(mum's method)
84 – 27 becomes 94 - 37

Jo’s method

Name	Age
Neil	66
Mandy	Mid 50s
Clare	50ish
Clare’s sister	50ish
Peter	51
Dawn	56
Stuart	49
Peter	30s
Michaela	25

Mum’s method

Name	Age
Mum	69
Gillian	61
Andy	Mid 50s
Trevor	70s

No idea

Name	Age
Tony	68
Bec	Late 20s
Tony	Mid 20s

The relative merits of the methods of teaching subtraction

Large scale studies in the 1930s

“One of the most debated questions in the teaching of arithmetic is that of the method to be used in subtraction”

This report recommended the complete abandonment of the decomposition method.

$$\begin{array}{rrr} & {}^{7}\cancel{8} & {}^{1}4 \\ - & 2 & 7 \\ \hline \end{array}$$

Decomposition

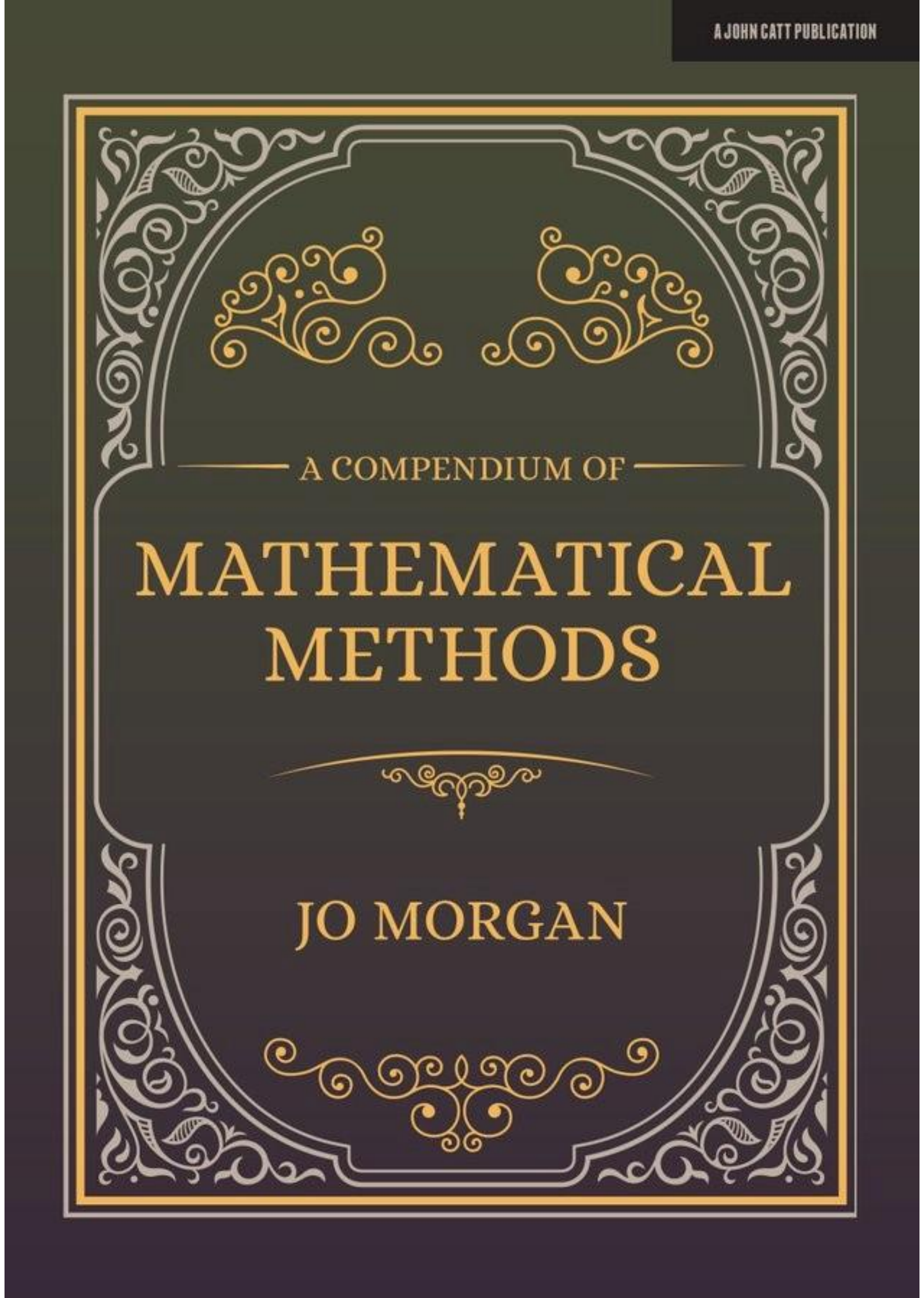
Re-write 84 as 70 + 14

$$\begin{array}{rrr} & 8 & {}^{1}4 \\ - & {}^{3}\cancel{2} & 7 \\ \hline \end{array}$$

Equal Addition

84 – 27 becomes 94 - 37

- Subtraction
- Multiplication
- Decimal multiplication
- Division
- Fraction division
- Highest common factor and lowest common multiple
- Reverse percentages
- Two-step linear equations
- Linear simultaneous equations
- Expanding double brackets
- Factorising non-monic quadratics
- Linear sequences
- Quadratic sequences
- Equations of linear graphs
- Quadratic inequalities
- Vertex of a quadratic
- Simplifying surds
- Angles in polygons
- Polynomial division

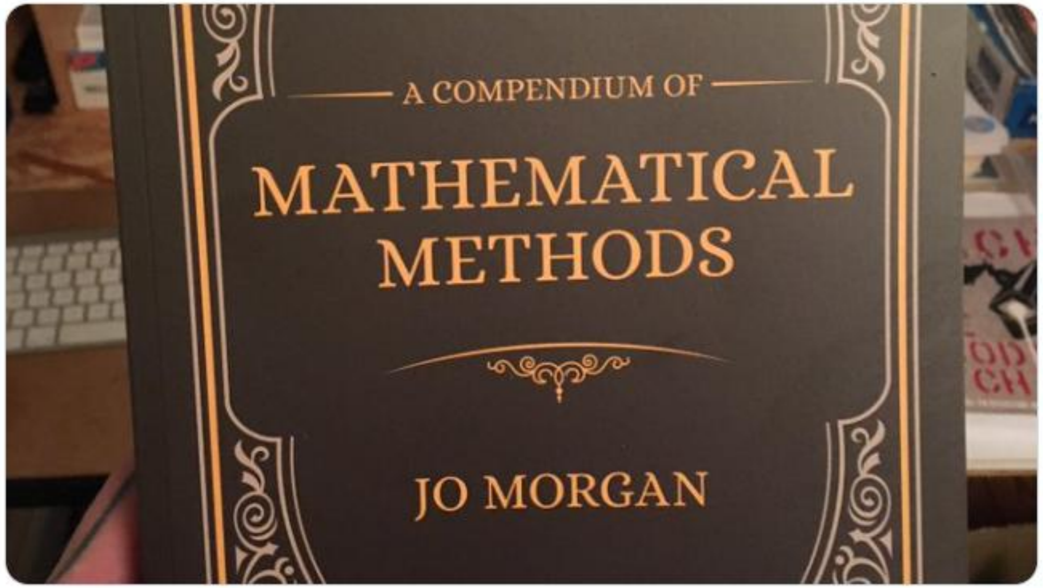


Peps
@PepsMccrea · Follow

My mum, an awesome maths teacher for c30 years, has just spent the afternoon reading @mathsjem new book and muttering...

What?
No way!
You can do it like that?!
Makes so much sense
Can't believe I didn't know about this

30 years. Because there was no book on methods. Until now.



8:11 PM · Dec 14, 2019

Let's Talk!

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