MEI®
Because maths matters!

Is the difference of two squares the most underutilised bit of maths?

15:00

Matt Ainscough

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How many of these calculations can you compute manually?

$$\frac{38^2-33^2}{42^2-29^2} \qquad \frac{35^2-3^2}{42^2-26^2}$$

$$\frac{38^2-35^2}{42^2-31^2} \qquad \frac{48^2-32^2}{42^2-26^2}$$

$$\frac{38^2-33^2}{40^2-35^2} \qquad \frac{36^2-35^2}{40^2-3^2}$$

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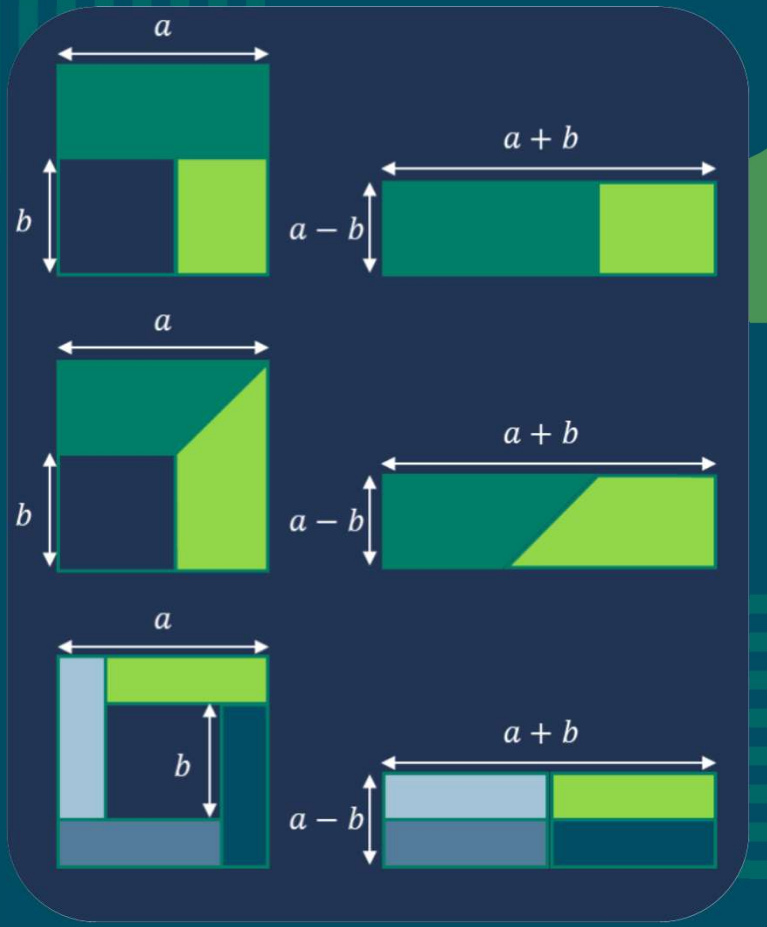

MATT AINSCOUGH

Local Lead

North West Region

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The case for the difference of two squares

Algebra

In addition to consolidating subject content from key stage 3, pupils should be taught to:

simplify and manipulate algebraic expressions (including those involving surds {and algebraic fractions}) by:

- factorising quadratic expressions of the form $x^2 + bx + c$, including the **difference of 2 squares**; {factorising quadratic expressions of the form $ax^2 + bx + c$}

KS4 – National Curriculum - Department for Education

When do you introduce the difference of two squares?

How else is it utilised in your curriculum?

Developing mathematical behaviours

Pattern spotting

Flexible thinking

Problem solving

Conjecturing

What's Possible?

Where could you go next with this activity?

Many numbers can be expressed as the difference of two perfect squares.

For example,

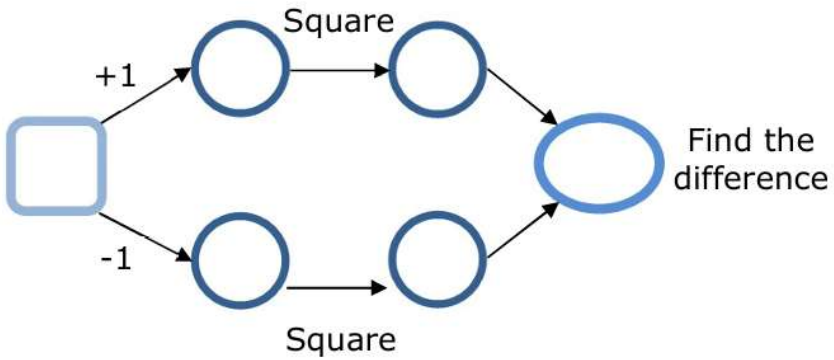
$$20 = 6^2 - 4^2$$
$$21 = 5^2 - 2^2$$
$$36 = 6^2 - 0^2$$

How many of the numbers from **1** to **30** can you express as the difference of two perfect squares?

Difference of Two Squares

Where could you go next with this activity?

Choose a number in the 3 times table. Take the numbers on either side of your chosen number and find the difference between their squares.



Try it a few times.
What do you notice?
Can you prove it will always happen?

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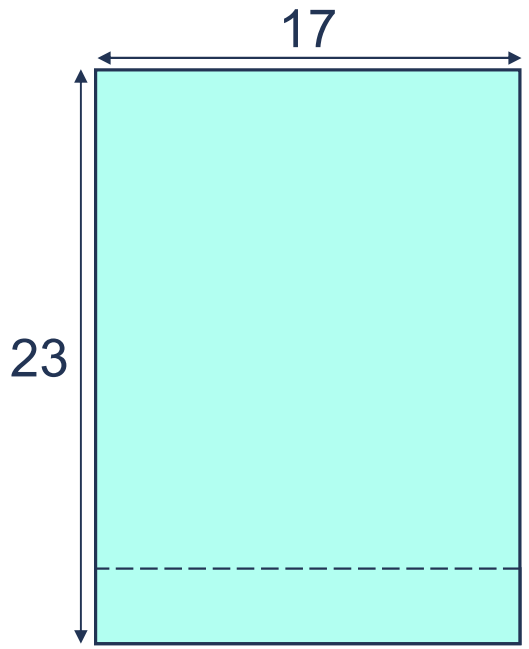
Product to difference

$$(a + b)(a - b) \rightarrow a^2 - b^2$$

Multiplication

Calculate 23×17

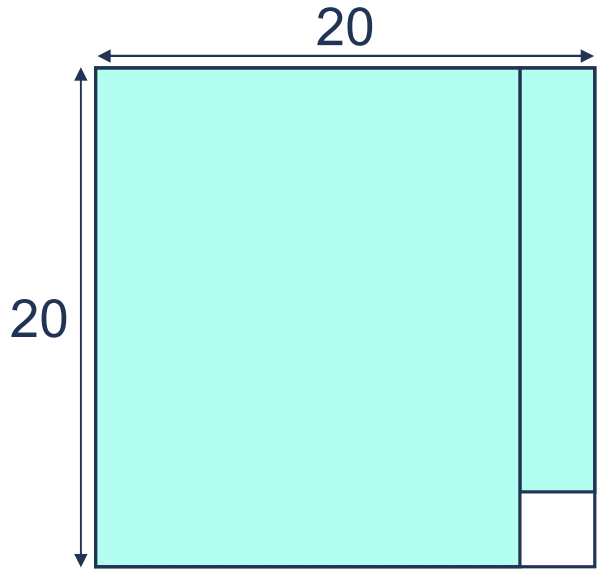
$23 \times 17 = (20 + 3)(20 - 3)$



Multiplication

Calculate 23×17

$$\begin{aligned} 23 \times 17 &= (20 + 3)(20 - 3) \\ &= 20^2 - 3^2 \\ &= 400 - 9 \\ &= 391 \end{aligned}$$



Try 24×16

Multiplication

Inspired by MEDIAN Don Steward mathematics teaching: two, 2-digit multiplications Task 2

17×15

12×20

9×23

18×14

Compare each of these calculations to $16^2 = 256$

Sowing seeds of thought …

Show that $(9+4)(9-4)=65$

$$9^2-4^2=81-16=65$$

Show that $\left(\sqrt{9}+\sqrt{4}\right)\left(\sqrt{9}-\sqrt{4}\right)=5$

$$(\sqrt{9})^2-(\sqrt{4})^2=9-4=5$$

Show that $\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)=2$

$$(\sqrt{3})^2-1^2=3-1=2$$

Show that $\left(2-\sqrt{5}\right)\left(\sqrt{5}+2\right)=-1$

$$\left(\sqrt{5}\right)^2-2^2=5-4=-1$$

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Difference to product

$$a^2 - b^2 \rightarrow (a + b)(a - b)$$

$99^2 - 1^2$ $(99 + 1)(99 - 1) = 100 \times 98 = 9800$

$47^2 - 3^2$ $(47 + 3)(47 - 3) = 50 \times 44 = 2200$

Where else could you take this?

$79^2 - 71^2$ $(79 + 71)(79 - 71) = 150 \times 8 = 1200$

$181^2 - 180^2$ $(181 + 180)(180 - 180) = 361 \times 1 = 361$

$99^2 - 1^2$ $\quad (99 + 1)(99 - 1) = 100 \times 98 = 9800$

$47^2 - 3^2$ $\quad (47 + 3)(47 - 3) = 50 \times 44 = 2200$

$$99^2 - 1^2 = (99 + 1)(99 - 1)$$
$$99^2 = (99 + 1)(99 - 1) + 1$$
$$99^2 = 100 \times 98 + 1$$
$$99^2 = 9800 + 1 = 9801$$

$$47^2 - 3^2 = (47 + 3)(47 - 3)$$
$$47^2 = (47 + 3)(47 - 3) + 3^2$$
$$47^2 = 50 \times 44 + 3^2$$
$$47^2 = 2200 + 9 = 2209$$

a nice number to multiply by

$$n^2 = (n + k)(n - k) + k^2$$

a nice number to multiply by

Calculating squares

$$n^2 = (n+k)(n-k) + k^2$$

a nice number to multiply by

a nice number to multiply by

Try:

101^2

26^2

53^2

71^2

$99^2 - 1^2$ $(99 + 1)(99 - 1) = 100 \times 98 = 9800$

$47^2 - 3^2$ $(47 + 3)(47 - 3) = 50 \times 44 = 2200$

Where else could you take this?

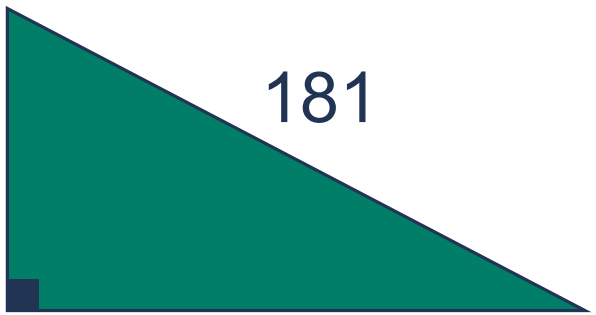
$79^2 - 71^2$ $(79 + 71)(79 - 71) = 150 \times 8 = 1200$

$181^2 - 180^2$ $(181 + 180)(180 - 180) = 361 \times 1 = 361$

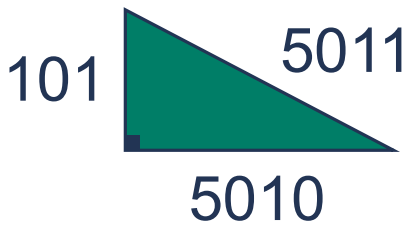
Pythagorean triples

$181^2 - 180^2$ $\quad (181 + 180)(180 - 180) = 361 \times 1 = 361$

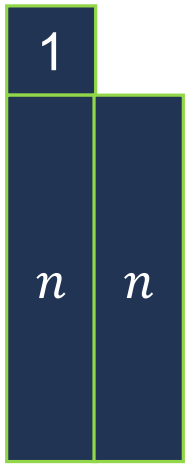
$\sqrt{361} = 19$



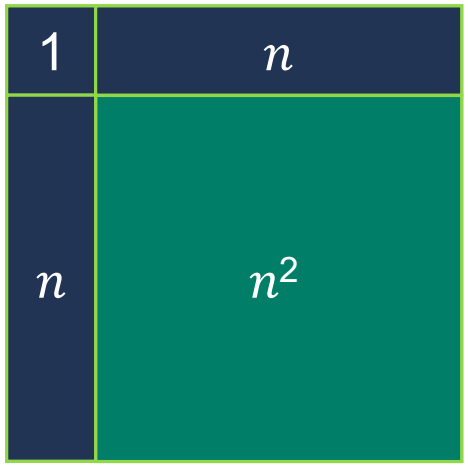
$101^2 = 10201$ $\quad \frac{10201}{2} = 5010.5$ $\quad 101^2 = 5011^2 - 5010^2$



Nearby squares

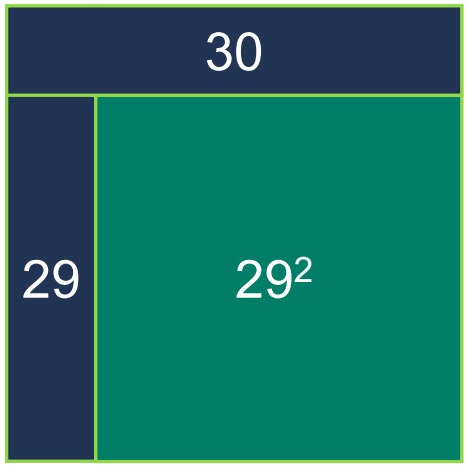


Nearby squares



$29^2 = \ ?$

Nearby squares



$29^2 = \ ?$

$30^2 = 29^2 + (29 + 30)$

$29^2 = 30^2 - (29 + 30) = 841$

Try:

39^2

71^2

51^2

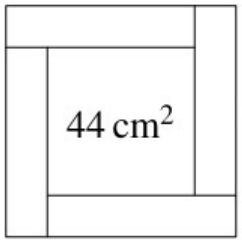
499^2

Problem solving

Problem #1

22. The diagram shows four congruent rectangles, each of perimeter 20 cm, surrounding a square of area 44 cm^2.
What is the area of each rectangle?

A 14 cm^2 B 16 cm^2 C 18 cm^2 D 20 cm^2 E 22 cm^2



[UKMT, JMC, 2021]

Problem #2

Find all pairs of positive integers x and y such that $x^2 - y^2 = 45$

Problem #3

Work out $\left(\sqrt{2024}+\sqrt{2026}\right)\left(\sqrt{2026}-\sqrt{2024}\right)$

Problem #4

22. The numbers x and y satisfy both of the equations

$$23x + 977y = 2023 \qquad \text{and} \qquad 977x + 23y = 2977.$$

What is the value of $x^2 - y^2$?

A 1 B 2 C 3 D 4 E 5

[UKMT, IMC, 2023]

Problem #5

4 is a square number and is one more than 3, a prime number.

Can you find another square number that is one more than a prime number?

could try $1+5=6\times$
$1+7=8\times$
$1+11=12\times$ etc

BUT if the square number is a^2
then looking for $a^2 = 1+p$
so prime $p = a^2-1 = (a-1)(a+1)$
which can't be prime unless $(a-1)=1$ $a=2$
which is the example above, so no others

Back to our starter

How many of these calculations can you compute manually?

$$\frac{38^2-33^2}{42^2-29^2}=\frac{(38+33)(38-33)}{(42+29)(42-29)}=\frac{71\times5}{71\times13}=\frac{5}{13}$$

$$\frac{35^2-33^2}{42^2-26^2}=\frac{(35+33)(35-33)}{(42+26)(42-26)}=\frac{68\times2}{68\times16}=\frac{1}{8}$$

$$\frac{38^2-35^2}{42^2-31^2}=\frac{(38+35)(38-35)}{(42+3\ )(42-3\ )}=\frac{73\times3}{73\times11}=\frac{3}{11}$$

$$\frac{48^2-32^2}{42^2-26^2}=\frac{(48+32)(48-32)}{(42+2\ )(42-2\ )}=\frac{80\times16}{68\times16}=\frac{80}{68}=1\frac{3}{17}$$

$$\frac{38^2-3\ ^2}{40^2-35^2}=\frac{(38+33)(38-3\ )}{(40+3\ )(40-35)}=\frac{71\times5}{75\times5}=\frac{71}{75}$$

$$\frac{36^2-35^2}{40^2-31^2}=\frac{(36+35)(36-3\ )}{(40+31)(40-31)}=\frac{71\times1}{71\times9}=\frac{1}{9}$$

Thank you

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