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A level Add-Ins

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A level Add-Ins

A selection of ideas – all quick and easy to teach - that are outside the A level Mathematics specification but complement the maths in A level and provide students with new insights and a bigger picture.

Equation of a line

Heron’s Formula

Avoiding awkward differentiation

Linear approximation

Tangent half-angle substitution

The cubic formula

Descartes’ Rule of Signs

Equation of a line

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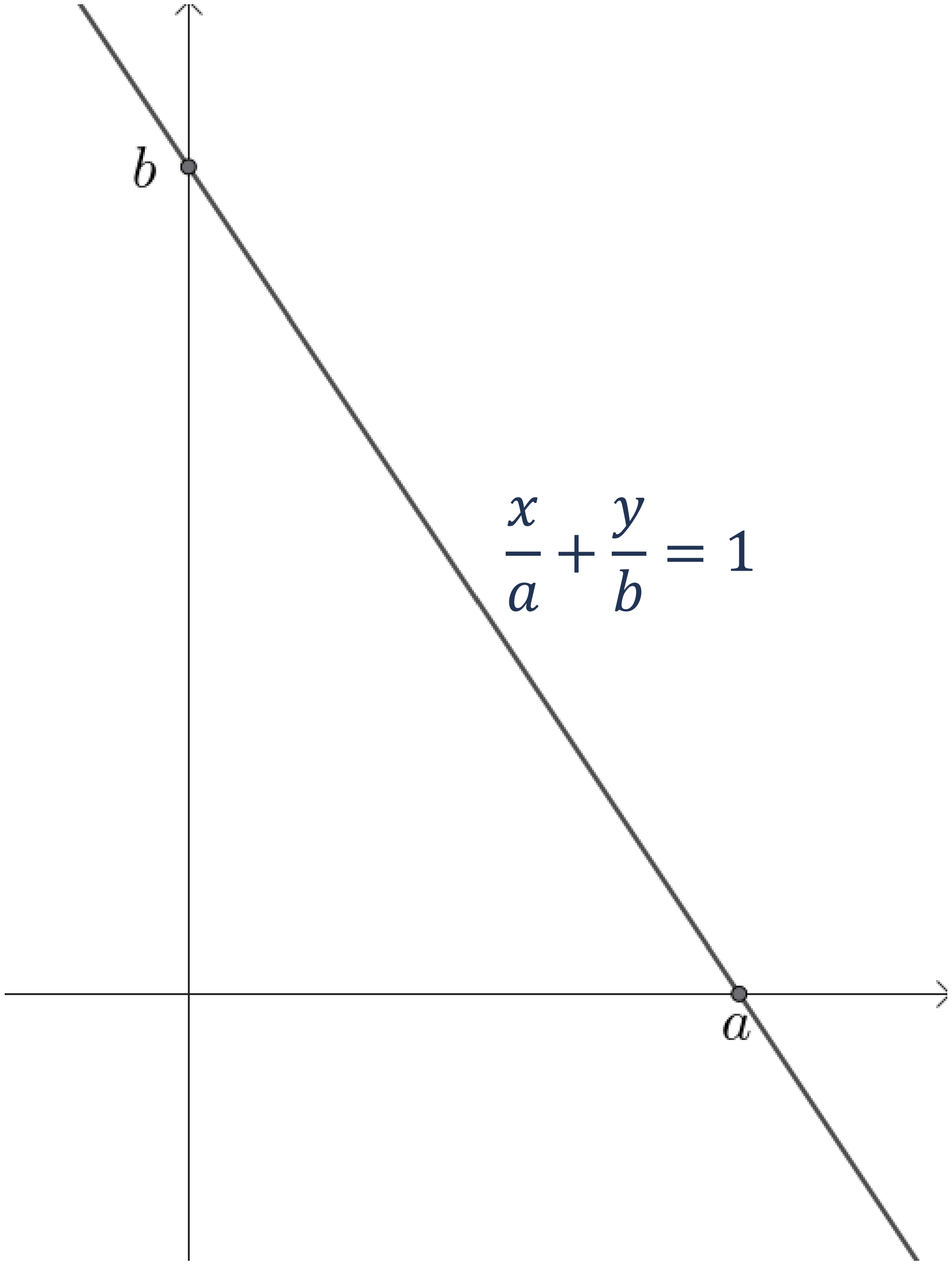


$$\frac{x}{a} + \frac{y}{b} = 1$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

AQA Paper 1, 2025

10 A researcher working for a frozen-food manufacturer uses the formula

$$\theta = 21 - Ae^{-kt}$$

to model the temperature of a dessert once it is taken out of a freezer.

In this model:

- θ is the temperature of the dessert in °C
- t is the time in hours since the dessert was removed from the freezer
- A and k are positive constants.

10 (a) Show how

$$\theta = 21 - Ae^{-kt}$$

can be rearranged to obtain

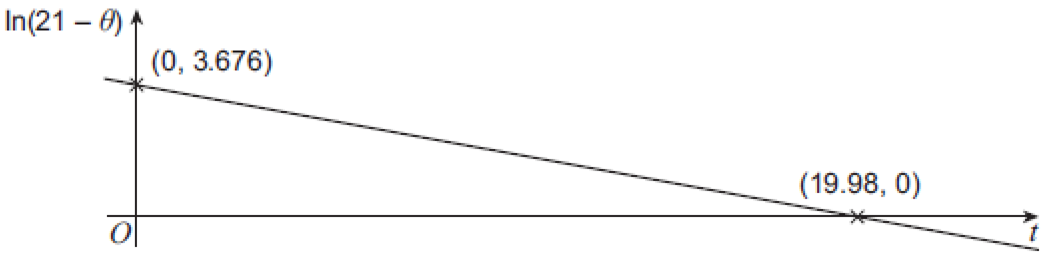
$$\ln(21 - \theta) = -kt + \ln A$$

[3 marks]

$$\frac{\ln(21-\theta)}{3.676} + \frac{t}{19.98} = 1$$

$$\ln(21-\theta) = -\frac{3.676}{19.98}t + 3.676$$

10 (b) The researcher uses measurements they have recorded to plot the graph of $\ln(21 - \theta)$ against t as shown in the diagram below.



10 (b) (i) Use the information on the graph to find the value of A

Give your answer to **three** significant figures.

[2 marks]

10 (b) (ii) Use the information on the graph to find the value of k

Give your answer to **three** significant figures.

[2 marks]

10 (b) (iii) Find the temperature of the dessert when it is initially removed from the freezer.

Give your answer to **three** significant figures.

[2 marks]

10 (c) The dessert is ready to be eaten when its temperature reaches 4 °C

Use the model to determine the time, after being removed from the freezer, for the dessert to reach this temperature.

Give your answer to the nearest 10 minutes.

[3 marks]

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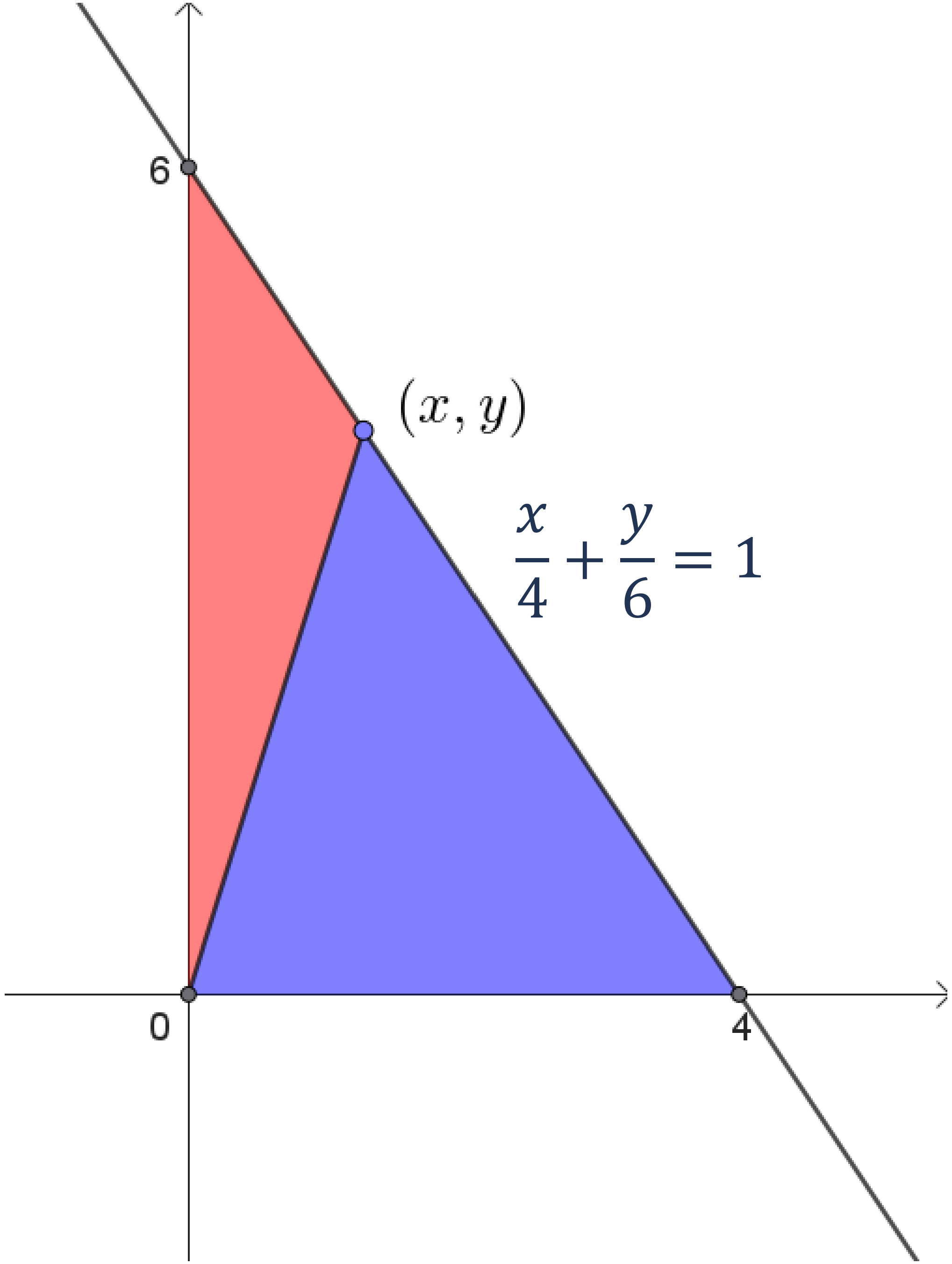
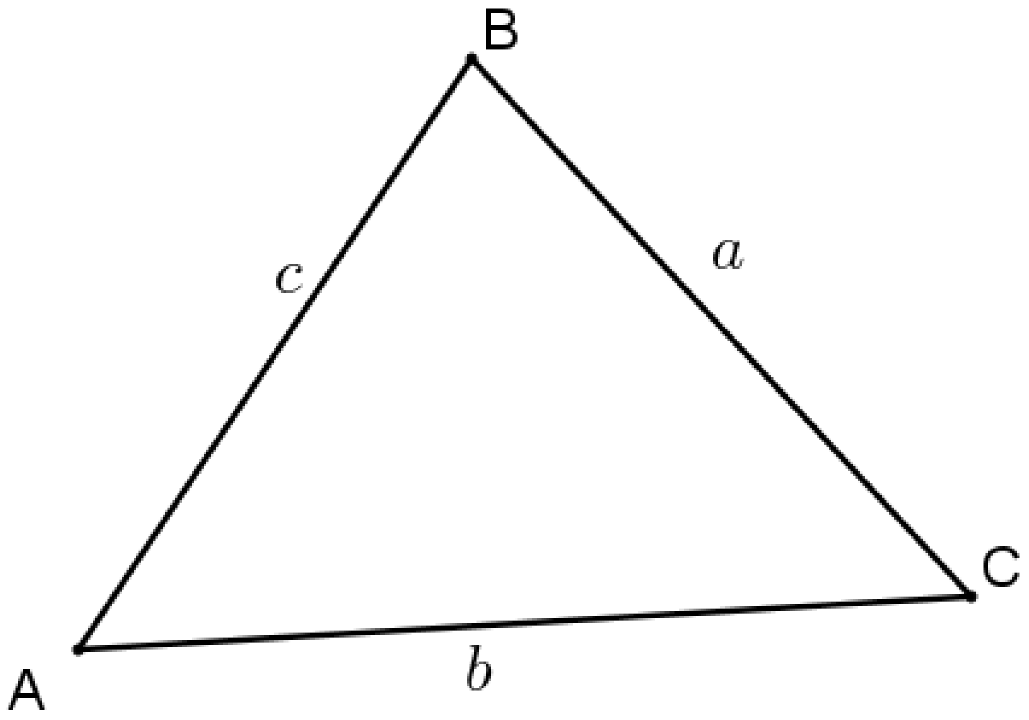


Red + Blue = Total

$$\frac{1}{2} \times 6 \times x + \frac{1}{2} \times 4 \times y = \frac{1}{2} \times 4 \times 6$$

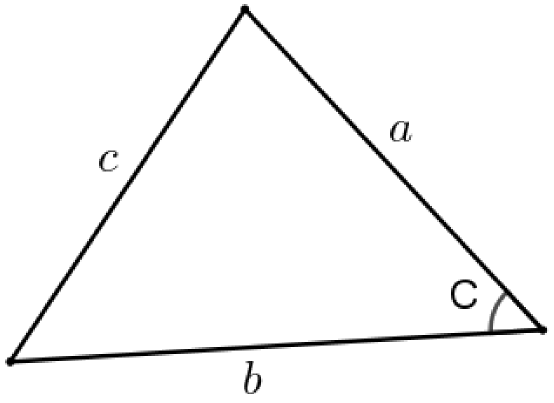
$$3x + 2y = 12$$

Heron’s Formula



$$s = \frac{a+b+c}{2}$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$



$$2abcosC = a^2 + b^2 - c^2$$

$$2absinC = 4\Delta$$

$$(2ab)^2 = (a^2 + b^2 - c^2)^2 + 16\Delta^2$$

$$(2ab + a^2 + b^2 - c^2)\big(2ab - (a^2 + b^2 - c^2)\big) = 16\Delta^2$$

$$\big((a+b)^2 - c^2\big)(c^2 - (a-b)^2) = 16\Delta^2$$

$$(a+b+c)(a+b-c)(c+a-b)(c-a+b) = 16\Delta^2$$

$$s = \frac{a+b+c}{2}$$

$$(2s)(2s-2c)(2s-2b)(2s-2a) = 16\Delta^2$$

$$\Delta^2 = s(s-c)(s-b)(s-a)$$

OCR(A) AS Paper 2, 2025

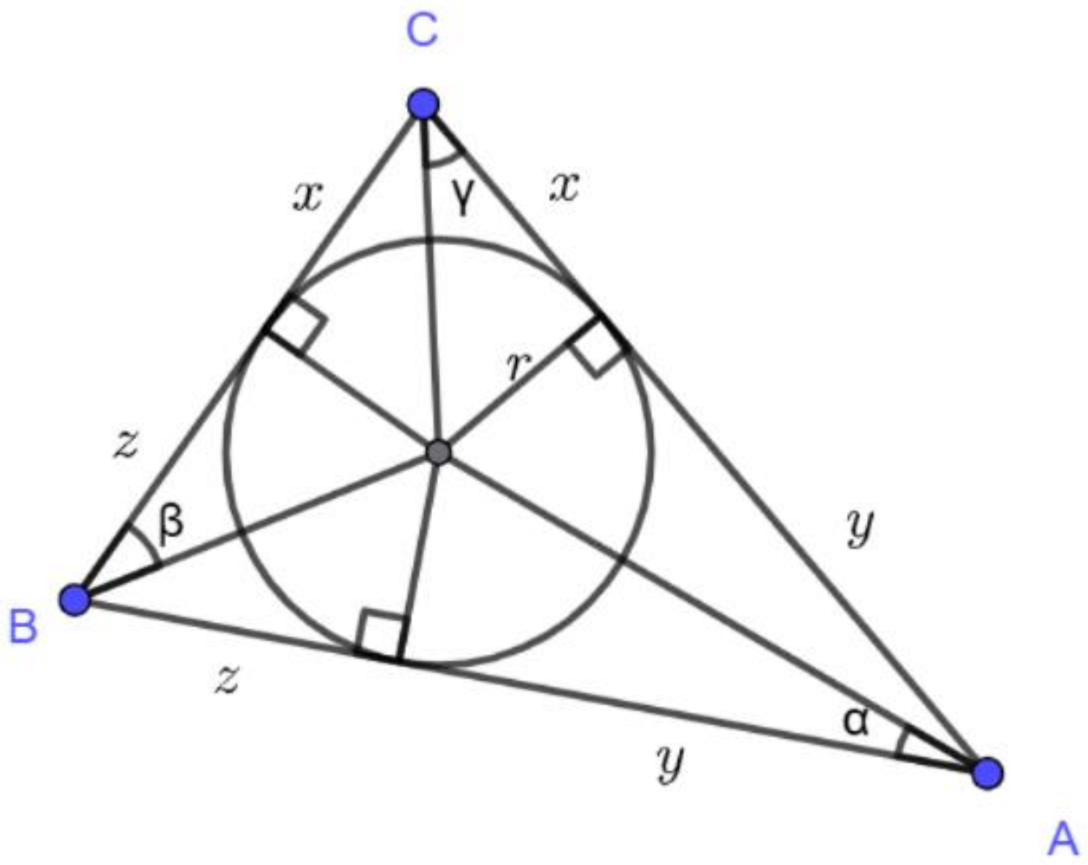
3 In a triangle ABC, $AB = 9\,\text{cm}$, $BC = 7\,\text{cm}$ and $AC = 4\,\text{cm}$.

(a) Show that $\cos CAB = \frac{2}{3}$. **[2]**

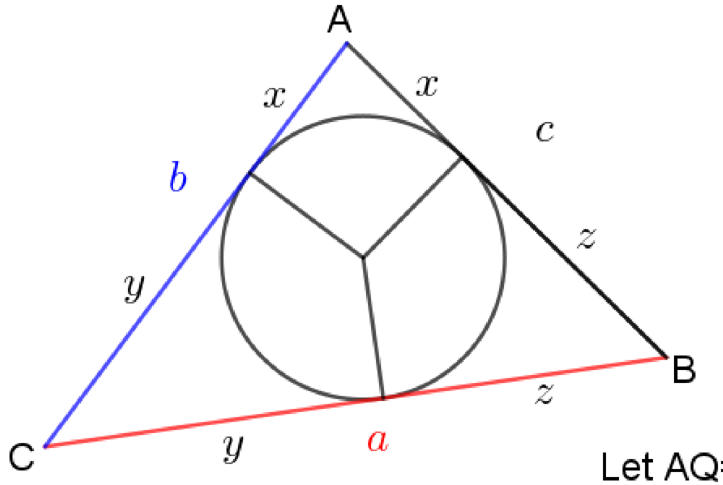
(b) Hence find the exact value of $\sin CAB$. **[2]**

(c) Find the exact area of triangle ABC. **[2]**

$$\Delta = \sqrt{s(s-c)(s-b)(s-a)} = \sqrt{10(10-9)(10-7)(10-4)} = 6\sqrt{5}$$



https://www.geogebra.org/m/u4zbvpjk



Let s be the semi perimeter of ABC

$$\Delta ABC = \frac{1}{2} r(a+b+c) = rs$$

$$2(x+y+z) = a+b+c = 2s$$

$$x = s-a; \quad y = s-c; \quad z = s-b$$

Let AQ=AS=d

$$CQ = d-b \Rightarrow CP = d-b$$

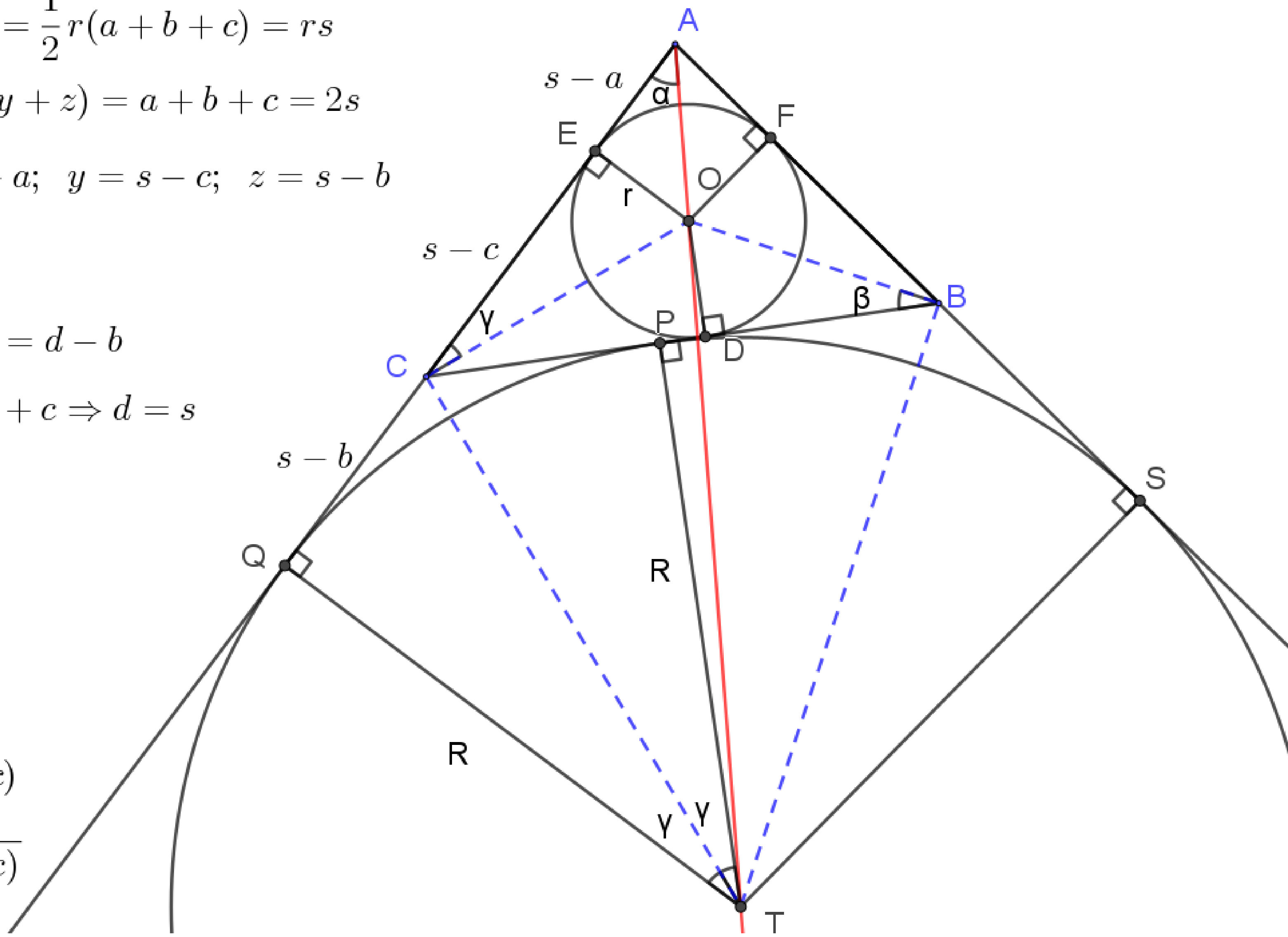
$$a = CB = CP + PB = d-b+d-c \Rightarrow 2d = a+b+c \Rightarrow d = s$$

$$\frac{r}{s-a} = \frac{R}{s} \qquad \frac{r}{s-c} = \frac{s-b}{R}$$

$$R = \frac{rs}{s-a} = \frac{(s-b)(s-c)}{r}$$

$$r^2 s^2 = s(s-a)(s-b)(s-c)$$

$$\Delta ABC = rs = \sqrt{s(s-a)(s-b)(s-c)}$$



https://www.geogebra.org/m/b6sbq8ys

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Avoiding awkward differentiation

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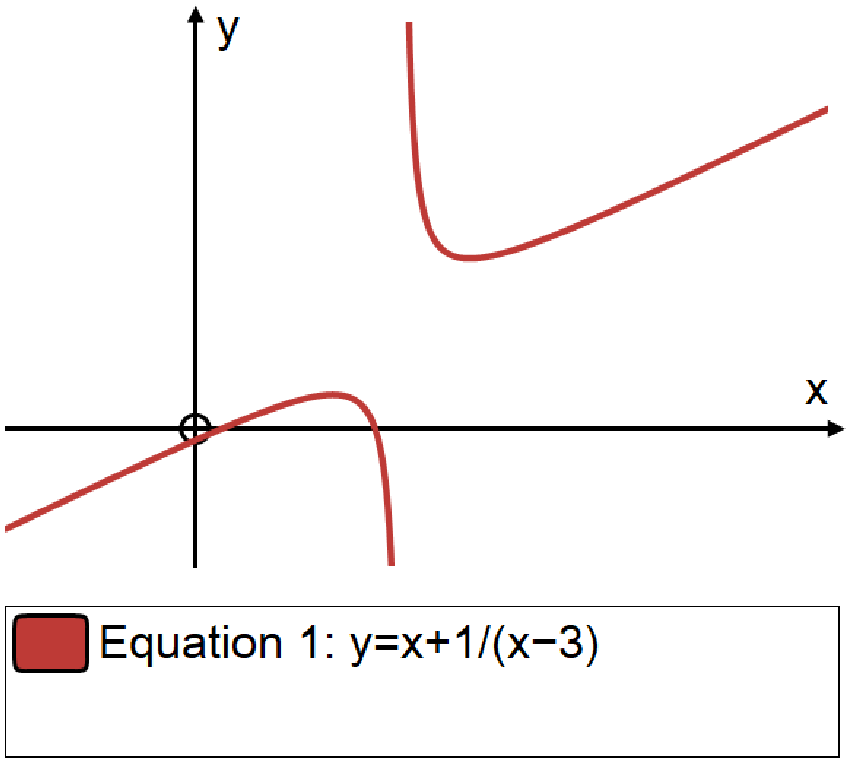
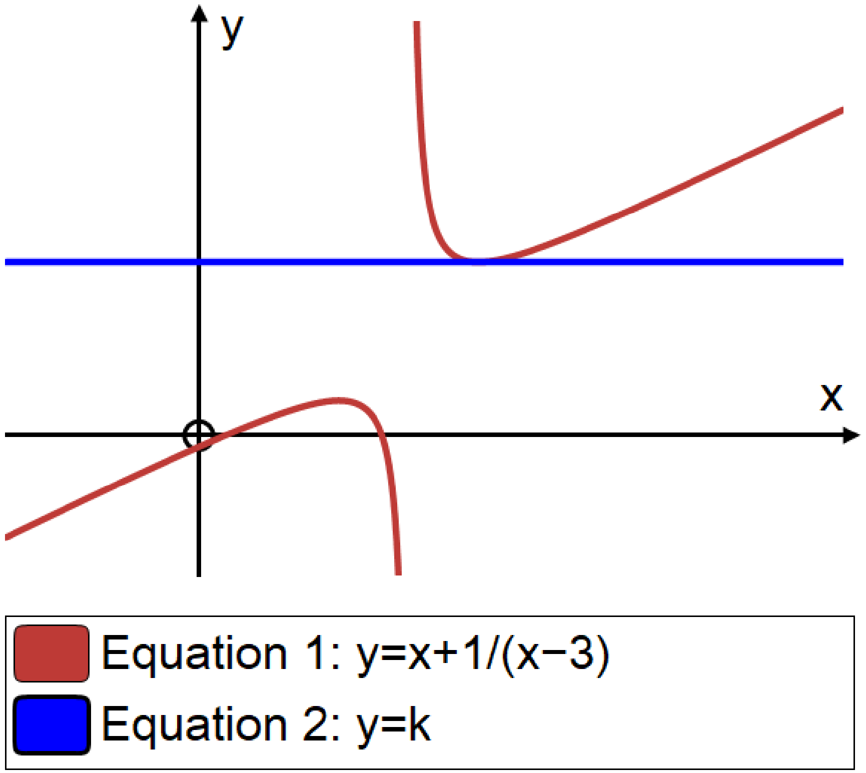

y
x
Equation 1: y=x+1/(x−3)

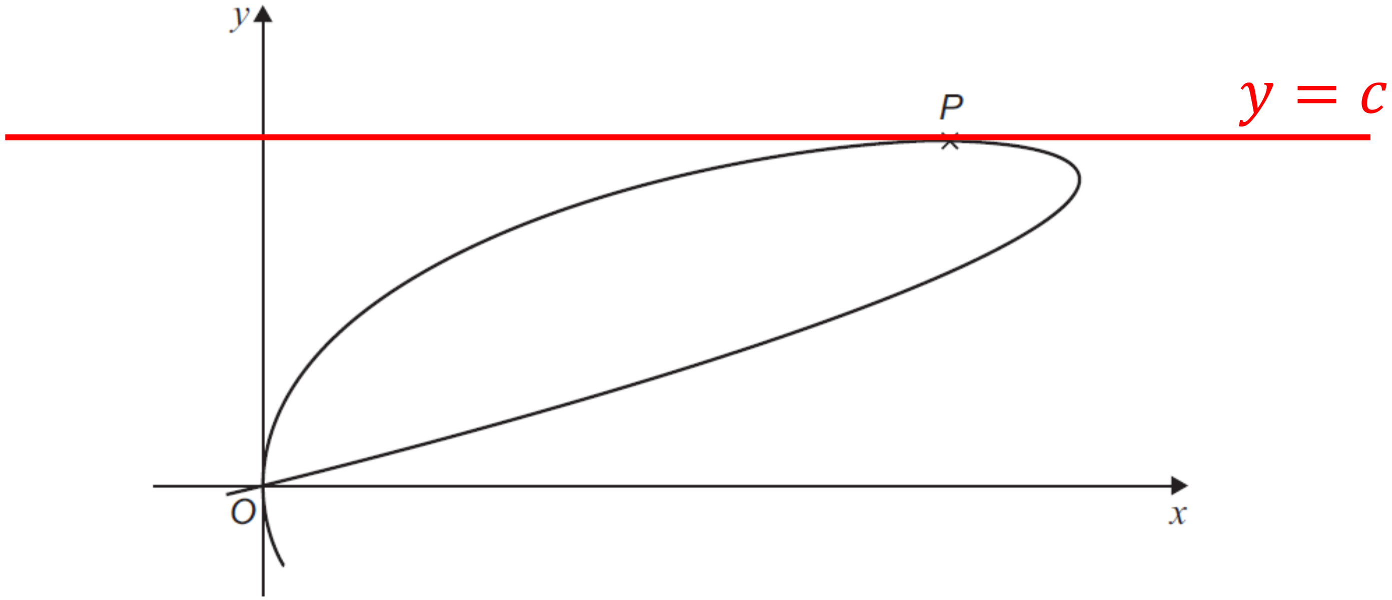

y
x
Equation 1: y=x+1/(x−3)
Equation 2: y=k

AQA Paper 1 2023

15 The curve with equation

$$x^2 + 2y^3 - 4xy = 0$$

has a single stationary point at P as shown in the diagram below.



$$x^2 - 4cx + 2c^3 = 0$$

$$16c^2 = 8c^3$$

$$8c^2(c-2) = 0$$

$$c = 0\,...!$$

$$c = 2: \qquad (x-4)^2 = 0 \Rightarrow (4,2)$$

15 (a) Show that the y-coordinate of P satisfies the equation

$$y^2(y-2) = 0$$

[7 marks]

15 (b) Hence, find the coordinates of P

[2 marks]

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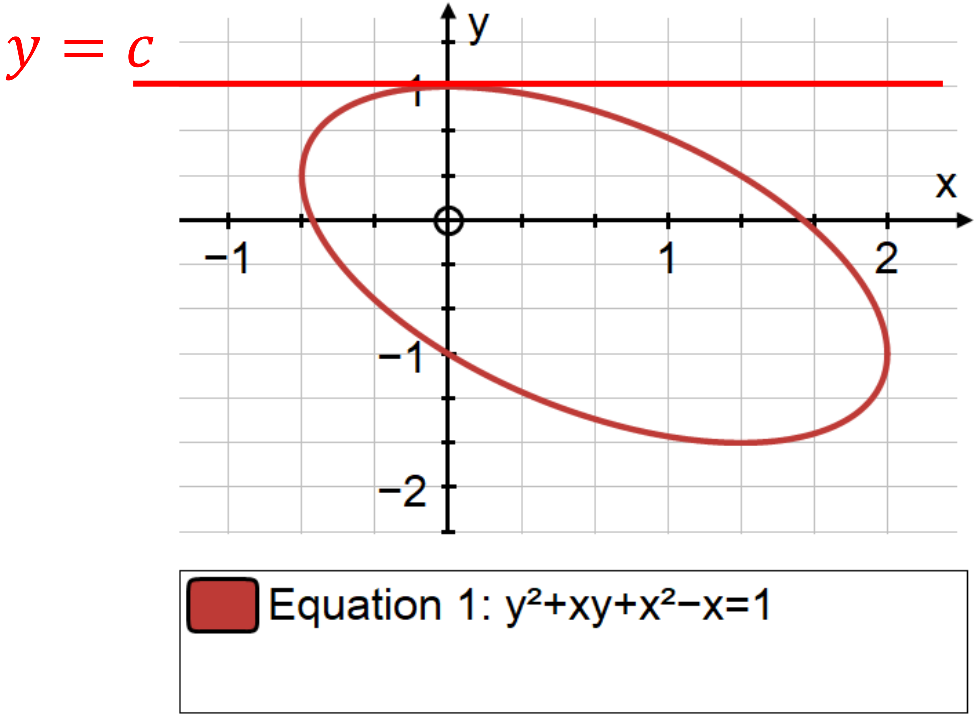
OCR(B) Paper 2 2024

13 Determine the coordinates of the turning points on the curve with equation

$$y^2 + xy + x^2 - x = 1. \qquad \textbf{[9]}$$

$$x^2 + (c-1)x + c^2 - 1 = 0$$



Repeated roots: $(c-1)^2 = 4(c^2-1)$

$c = 1$ $\qquad$ $c - 1 = 4(c+1) \Rightarrow c = -\frac{5}{3}$

$x = 0$: $(0,1)$ $\qquad$ $\left(x - \frac{4}{3}\right)^2 = 0$: $\left(\frac{4}{3}, -\frac{5}{3}\right)$

Edexcel Paper 1 2025

(part qn)

$$c = \frac{1 - x^2}{(1 + x^2)^2}$$

$$c(x^2)^2 + (2c + 1)x^2 + (c - 1) = 0$$

$$(2c + 1)^2 = 4c(c - 1)$$

$$4c + 1 = -4c$$

$$c = -\frac{1}{8}$$

15.

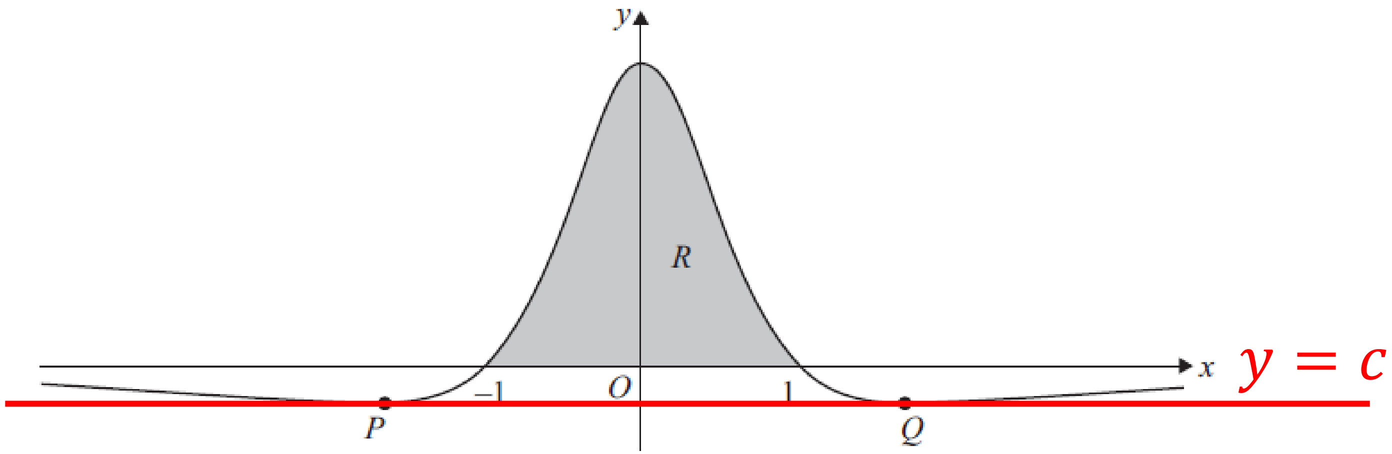


Figure 5

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 5 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1 - x^2}{(1 + x^2)^2}$$

The curve

- intersects the x-axis at -1 and 1
- has minimum turning points at P and Q

as shown in Figure 5.

(a) Use calculus to find the exact coordinates of P.

(5)

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AQA Paper 3 2022

$$c = \frac{x^2 + 10}{2x + 5}$$

$$x^2 - 2cx + 10 - 5c = 0$$

$$4c^2 = 4(10 - 5c)$$

$$c^2 + 5c - 10 = 0$$

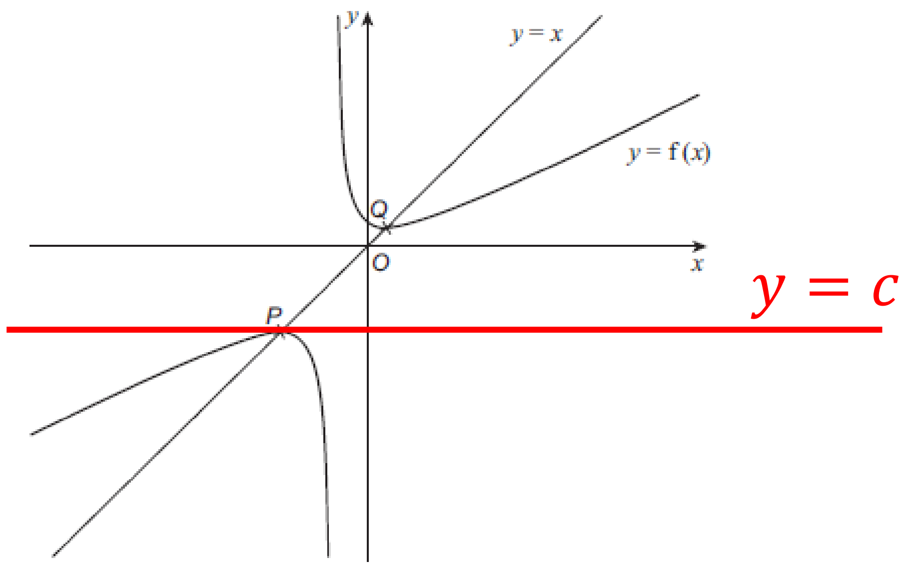
$$c = \frac{-5 \pm \sqrt{65}}{2}$$

10 The function f is defined by

$$f(x) = \frac{x^2 + 10}{2x + 5}$$

where f has its maximum possible domain.

The curve $y = f(x)$ intersects the line $y = x$ at the points P and Q as shown below.



10 (a) State the value of x which is not in the domain of f. **[1 mark]**

10 (b) Explain how you know that the function f is many-to-one. **[2 marks]**

10 (c) (i) Show that the x-coordinates of P and Q satisfy the equation

$$x^2 + 5x - 10 = 0$$

[2 marks]

10 (c) (ii) Hence, find the exact x-coordinate of P and the exact x-coordinate of Q. **[1 mark]**

10 (d) Show that P and Q are stationary points of the curve.

Fully justify your answer. **[5 marks]**

10 (e) Using set notation, state the range of f. **[2 marks]**

Linear approximation

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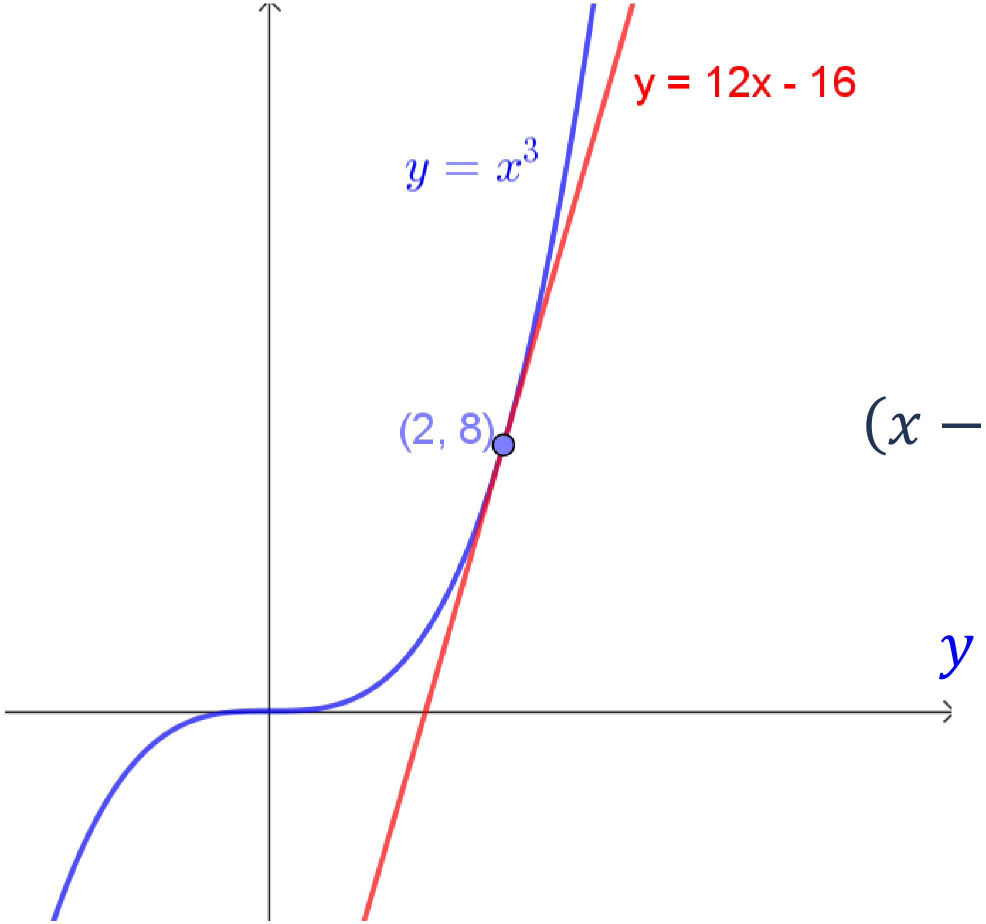
Linear Approximation



$$(x-2)^3 = x^3 - 6x^2 + 12x - 8$$

$$6(x-2)^2 = 6x^2 - 24x + 24$$

$$(x-2)^3 + 6(x-2)^2 = x^3 - 12x + 16$$

$$y = x^3 = (x-2)^3 + 6(x-2)^2 + 12x - 16$$

OCR(B) AS Paper 2, 2025

11 A circle has centre (4, –1) and radius 13.

(a) Write down the equation of the circle. **[2]**

(b) Determine the equation of the tangent to the circle at the point (9, 11). **[4]**

$$(x-4)^2+(y+1)^2=13^2$$

$$x^2-8x+y^2+2y=152$$

$$(x-9)^2+10x-81+(y-11)^2+24y-121=152$$

$$(x-9)^2+(y-11)^2+10x+24y=354$$

Tangent half-angle substitution

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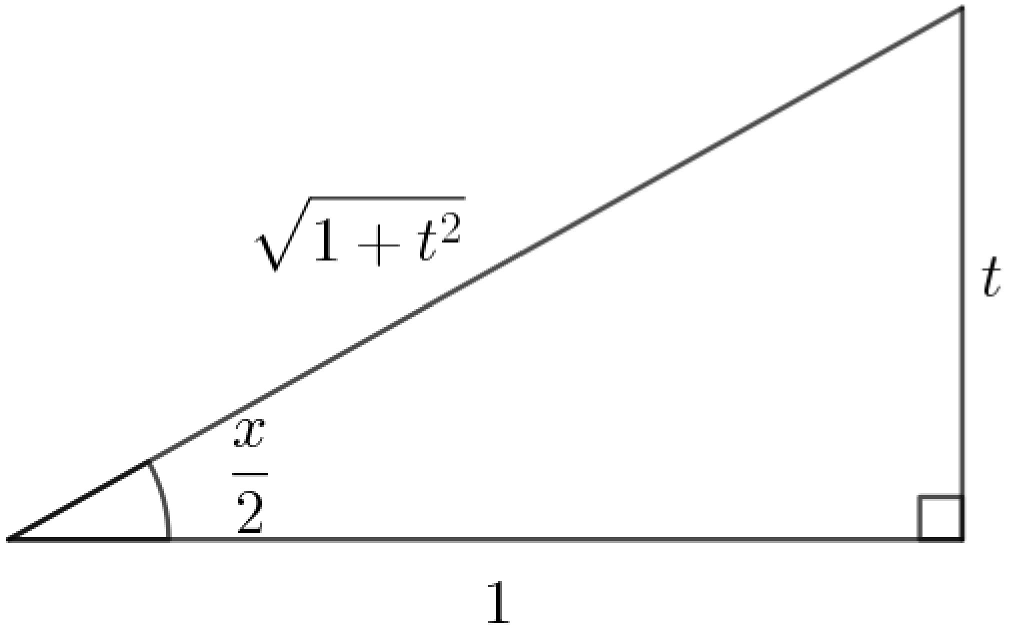
t=tan(x/2) substitution



$$\sin\left(\frac{x}{2}\right) = \frac{t}{\sqrt{1+t^2}}, \quad \cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+t^2}}$$

$$\sin(x) = \frac{2t}{1+t^2}, \quad \cos(x) = \frac{1-t^2}{1+t^2}$$

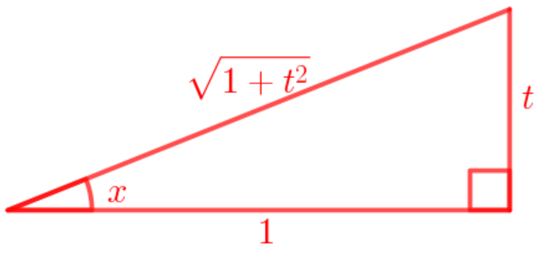
$$t = \tan\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right) = \frac{1}{2}(1+t^2)$$

OCR(A) Paper 1, 2025

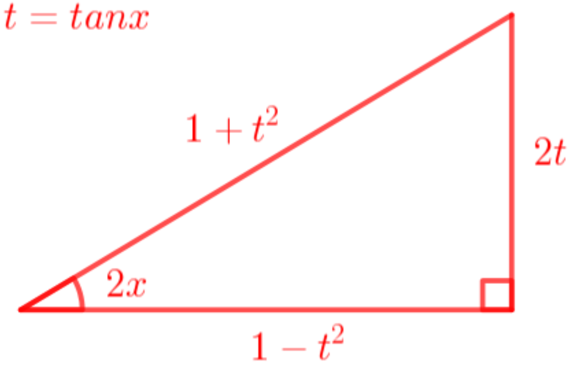
9 **(a)** Express $3\cos 2x + 4\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(b) **In this question you must show detailed reasoning.**

Solve the equation $\dfrac{3\cos 2x + 4\sin 2x + 6}{3\cos 2x + 4\sin 2x - 1} = 4$ for $0° < x < 360°$. **[7]**



$t = tanx$



$$\frac{3 - 3t^2 + 8t + 6 + 6t^2}{3 - 3t^2 + 8t - 1 - t^2} = 4$$

$$19t^2 - 24t + 1 = 0$$

$$t = tanx = \frac{12 \pm 5\sqrt{5}}{19}$$

Example

$$\int \frac{1}{\cos x}\,\mathrm{d}x$$

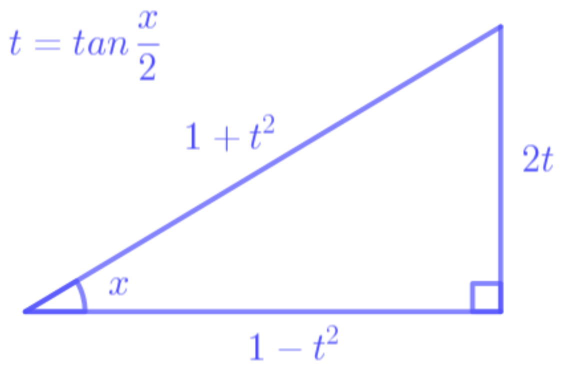
$$= \int \frac{1}{\left(\frac{1-t^2}{1+t^2}\right)} \times \frac{2}{1+t^2}\,\mathrm{d}t \;= \int \frac{2}{1-t^2}\,\mathrm{d}t$$

$$= \int \frac{1}{1+t} + \frac{1}{1-t}\,\mathrm{d}t$$

$$= \ln|1+t| - \ln|1-t| + c$$

$$= \ln\left|\frac{1+t}{1-t}\right| + c$$

$$= \ln\left|\frac{1+\tan\left(\frac{x}{2}\right)}{1-\tan\left(\frac{x}{2}\right)}\right| + c$$



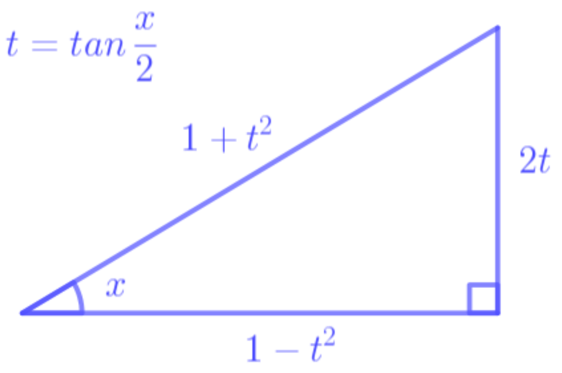
$$= \ln\left|\frac{(1+t)^2}{1-t^2}\right| + c$$

$$= \ln\left|\frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}\right| + c$$

$$= \ln|\sec x + \tan x| + c$$

Example

$$\int \frac{1}{\sin x}\,\mathrm{d}x$$



$$= \int \frac{1}{\left(\frac{2t}{1+t^2}\right)} \times \frac{2}{1+t^2}\,\mathrm{d}t$$

$$= \int \frac{1}{t}\,\mathrm{d}t$$

$$= \ln|t| + c$$

$$= \ln\left|\frac{(1+t^2)-(1-t^2)}{2t}\right| + c$$

$$= \ln\left|\tan\left(\frac{x}{2}\right)\right| + c$$

$$= \ln|\operatorname{cosec} x - \cot x| + c$$

The cubic formula

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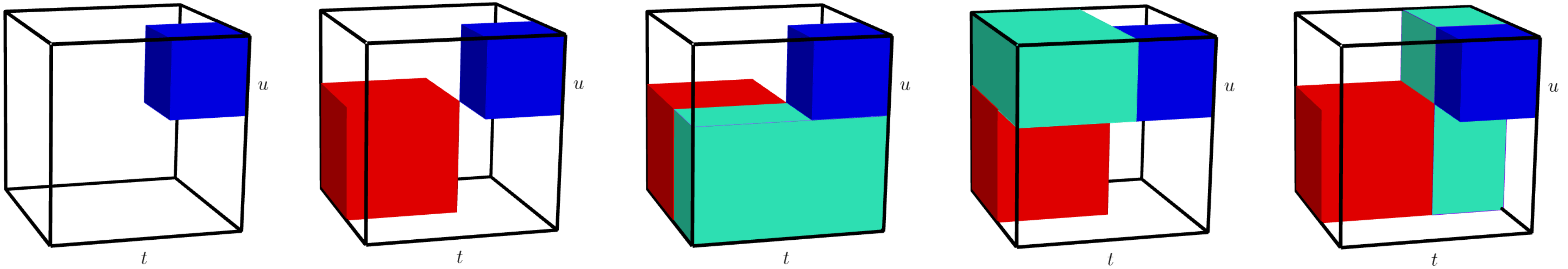
The cubic formula

$$x^3 + mx = n \quad \Leftarrow \quad x = \sqrt[3]{\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}} - \sqrt[3]{-\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}}$$



$$t^3 \equiv u^3 + (t-u)^3 + 3tu(t-u)$$

$$(t-u)^3 + 3tu(t-u) \equiv t^3 - u^3$$

$$x^3 \quad + \quad mx \quad = \quad n$$

$$m = 3tu$$
$$n = t^3 - u^3$$

$$n = t^3 - \left(\frac{m}{3t}\right)^3$$

$$(t^3)^2 - n(t^3) - \left(\frac{m}{3}\right)^3 = 0$$

$$t^3 = \frac{n + \sqrt{n^2 + 4\left(\frac{m}{3}\right)^3}}{2} = \frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}$$

$$u^3 = -\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}$$

$$x = t - u = \sqrt[3]{\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}} - \sqrt[3]{-\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}$$

Example

$$x^3 + 3x^2 + 15x - 17 = 0$$

$$(x+1)^3 = x^3 + 3x^2 + 3x + 1$$

$$(x+1)^3 + 12(x+1) - 30 = 0$$

$$X^3 + 12X = 30$$

$$x = \sqrt[3]{\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}} - \sqrt[3]{-\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}$$

$$X = \sqrt[3]{15 + \sqrt{225 + 64}} - \sqrt[3]{-15 + \sqrt{225 + 64}}$$

$$X = \sqrt[3]{32} - \sqrt[3]{2}$$

$$x = \sqrt[3]{32} - \sqrt[3]{2} - 1$$

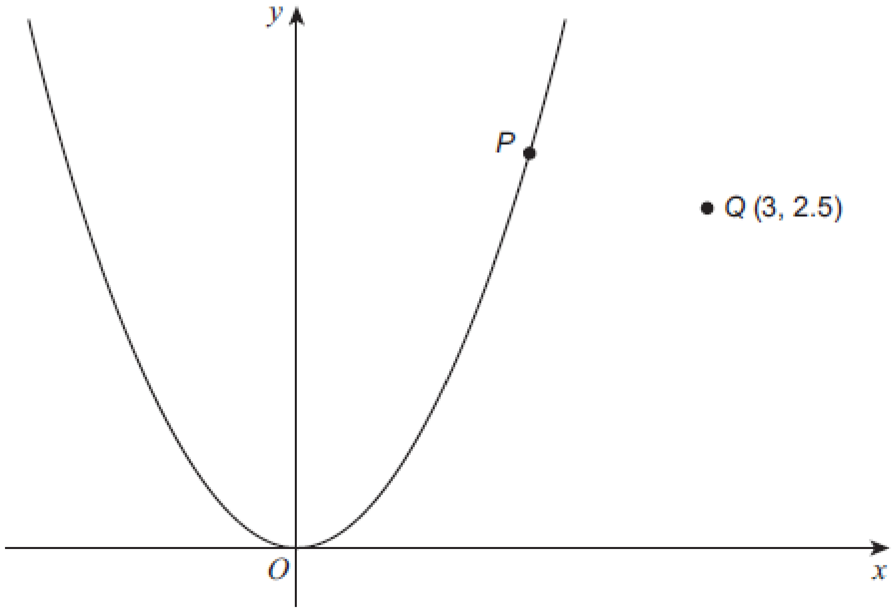
AQA Paper 1, 2025

15 A curve has equation

$$y = x^2$$

The point Q has coordinates (3, 2.5)

The point P on the curve which is closest to the point Q is shown on the diagram below.



15 (a) Show that the x-coordinate of P satisfies the equation

$$2x^3 - 4x - 3 = 0$$

[4 marks]

$$x = \sqrt[3]{\frac{27+\sqrt{345}}{36}} - \sqrt[3]{\frac{-27+\sqrt{345}}{36}}$$

$$x = \sqrt[3]{\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}} - \sqrt[3]{-\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}$$

15 (b) The Newton–Raphson method is to be used to find an approximate solution to the equation

$$2x^3 - 4x - 3 = 0$$

Show that the Newton–Raphson method generates the iterative formula

$$x_{n+1} = \frac{4x_n^3 + 3}{6x_n^2 - 4}$$

[4 marks]

15 (c) Starting with $x_0 = 3$, use the iterative formula given in part **(b)** to find the value of x_3

Give your answer to **three** decimal places.

[2 marks]

15 (d) Hence find the distance PQ

Give your answer to **two** decimal places.

[2 marks]

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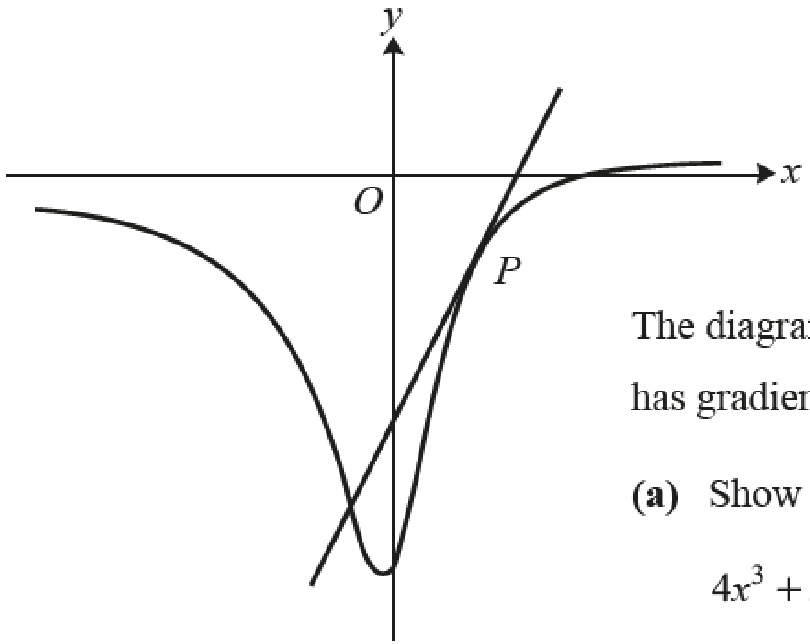
OCR(A) Paper 3, 2022

5 In this question you must show detailed reasoning.



$$x = \sqrt[3]{\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}} - \sqrt[3]{-\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}$$

The diagram shows the curve with equation $y = \frac{2x-3}{4x^2+1}$. The tangent to the curve at the point P has gradient 2.

(a) Show that the x-coordinate of P satisfies the equation

$$4x^3 + 3x - 3 = 0.$$

[5]

$$x^3 + \frac{3}{4}x = \frac{3}{4}$$

(b) Show by calculation that the x-coordinate of P lies between 0.5 and 1. **[2]**

(c) Show that the iteration

$$x_{n+1} = \frac{3 - 4x_n^3}{3}$$

cannot converge to the x-coordinate of P whatever starting value is used. **[2]**

$$x = \frac{1}{2}\left(\sqrt[3]{3 + \sqrt{10}} - \sqrt[3]{-3 + \sqrt{10}}\right)$$

(d) Use the Newton-Raphson method, with initial value 0.5, to determine the coordinates of P correct to **5** decimal places. **[5]**

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OCR(A) Paper 2, 2024

$$x = \frac{1}{2}\left(\sqrt[3]{\frac{1+i}{\sqrt{2}}} - \sqrt[3]{\frac{-1+i}{\sqrt{2}}}\right) = \frac{1}{2}\left(\frac{i-1}{\sqrt{2}} - \frac{1+i}{\sqrt{2}}\right) = -\frac{\sqrt{2}}{2}$$

6 **In this question you must show detailed reasoning.**

(a) **(i)** Use the formula for $\cos(A+B)$, and the double angle formulae, to show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **[2]**

(ii) Use this result to solve the equation $4\cos^3\theta - 3\cos\theta - \frac{\sqrt{2}}{2} = 0$ for $0° \leqslant \theta \leqslant 180°$. **[3]**

(b) **(i)** Show that $\left(x + \frac{\sqrt{2}}{2}\right)\left(4x^2 - 2\sqrt{2}x - 1\right) = 4x^3 - 3x - \frac{\sqrt{2}}{2}$. **[1]**

(ii) Hence find the exact roots of the equation $4x^3 - 3x - \frac{\sqrt{2}}{2} = 0$. **[2]**

(c) Use the results from parts **(a)(ii)** and **(b)(ii)** to show that $\cos 15° = \frac{\sqrt{2}+\sqrt{6}}{4}$. **[2]**

Descartes’ Rule of Signs

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Special case of Descartes Rule of Signs

If the number of sign changes in the polynomial, P(x), is 0 or 1 then this (i.e. 0 or 1) is the number of positive roots of P(x)=0.

If the number of sign changes in the polynomial, P(-x), is 0 or 1 then this (i.e. 0 or 1) is the number of negative roots of P(x)=0.

e.g. $x^4 - 3x - 4 = 0$ must have exactly one positive root and one negative root.

$x^3 + 3x - 4 = 0$ must have exactly one positive root and no negative roots.

OCR(A) AS Paper 2, 2025

4 The cubic polynomial $2x^3 - kx^2 + 4x + k$, where k is a constant, is denoted by $f(x)$.
It is given that $f'(2) = 16$.

(a) Show that $k = 3$. **[4]**

For the remainder of the question, you should use this value of k.

(b) Use the factor theorem to show that $(2x+1)$ is a factor of $f(x)$. **[2]**

(c) Hence show that the equation $f(x) = 0$ has only **one** real root. **[4]**

OCR(B) Paper 3, 2023

4 **In this question you must show detailed reasoning.**

Find the coordinates of the points where the curve $y = x^3 - 2x^2 - 5x + 6$ crosses the x-axis. **[4]**

OCR(B) Paper 2, 2022

16 The equation of a curve is

$$y = 6x^4 + 8x^3 - 21x^2 + 12x - 6.$$

(a) **In this question you must show detailed reasoning.**

Determine

- The coordinates of the stationary points on the curve.
- The nature of the stationary points on the curve.
- The x-coordinate of the non-stationary point of inflection on the curve. **[12]**

(b) On the axes in the Printed Answer Booklet, sketch the curve whose equation is

$$y = 6x^4 + 8x^3 - 21x^2 + 12x - 6.$$ **[3]**